

# An equivariant foamy $\mathfrak{sl}_N$ -homology

Louis-Hadrien Robert

Emmanuel Wagner



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<http://www.math.uni-hamburg.de/home/robert/bhktalk.pdf>

$$\left\langle \begin{array}{c} \nearrow m \\ \searrow n \end{array} \right\rangle = \sum_{k=\max(0, m-n)}^m (-1)^{m-k} q^{k-m} \left\langle \begin{array}{c} \nearrow n+k-m \\ \searrow m-k \\ \nwarrow n \\ \swarrow k \end{array} \right\rangle$$

$$\left\langle \begin{array}{c} \nwarrow m \\ \swarrow n \end{array} \right\rangle = \sum_{k=\max(0, m-n)}^m (-1)^{m-k} q^{m-k} \left\langle \begin{array}{c} \nearrow n+k-m \\ \searrow m-k \\ \nwarrow n \\ \swarrow k \end{array} \right\rangle$$

$$\left\langle \left( \begin{array}{c} \text{circle with arrow } k \end{array} \right) \right\rangle = \left[ \begin{array}{c} N \\ k \end{array} \right]_q$$

$$\left\langle \begin{array}{c} m+n \xrightarrow{m} \\ \xrightarrow{m} n \end{array} \right\rangle = \left[ \begin{array}{c} N-m \\ n \end{array} \right]_q \left\langle \begin{array}{c} \uparrow \\ m \end{array} \right\rangle$$

$$\left\langle \begin{array}{c} i \quad j \quad k \\ \swarrow \quad \searrow \quad \nearrow \\ \quad \quad \quad \nearrow \\ \quad \quad \quad j+k \\ \quad \quad \quad \uparrow \\ i+j+k \end{array} \right\rangle = \left\langle \begin{array}{c} i \quad j \quad k \\ \swarrow \quad \searrow \quad \nearrow \\ \quad \quad \quad \nearrow \\ \quad \quad \quad i+j \\ \quad \quad \quad \uparrow \\ i+j+k \end{array} \right\rangle$$

$$\left\langle \begin{array}{c} m+n \xrightarrow{m} \\ \xrightarrow{m} n \end{array} \right\rangle = \left[ \begin{array}{c} m+n \\ m \end{array} \right]_q \left\langle \begin{array}{c} \uparrow \\ m+n \end{array} \right\rangle$$

$$\left\langle \begin{array}{c} 1 \quad m \\ \uparrow m+1 \quad \downarrow m+1 \\ \leftarrow \quad \rightarrow \\ \uparrow m+1 \quad \downarrow m+1 \\ 1 \quad m \end{array} \right\rangle = \left\langle \begin{array}{c} \uparrow \\ 1 \end{array} \right\rangle \left\langle \begin{array}{c} \downarrow \\ m \end{array} \right\rangle + [N-m-1]_q \left\langle \begin{array}{c} 1 \quad m \\ \swarrow \quad \searrow \\ \quad \quad \quad \nearrow \\ \quad \quad \quad m-1 \\ \quad \quad \quad \uparrow \\ 1 \quad m \end{array} \right\rangle$$

$$\left\langle \begin{array}{c} m \quad n+l \\ \uparrow n+k \quad \uparrow m+l-k \\ \leftarrow n+k-m \quad \rightarrow k \\ \uparrow n \quad \uparrow m+l \end{array} \right\rangle = \sum_{j=\max(0, m-n)}^m \left[ \begin{array}{c} l \\ k-j \end{array} \right]_q \left\langle \begin{array}{c} m \quad n+l \\ \uparrow m-j \quad \uparrow n+l+j \\ \leftarrow j \quad \rightarrow n+j-m \\ \uparrow n \quad \uparrow m+l \end{array} \right\rangle$$

$$\langle \langle \text{circle with arrow } k \rangle \rangle = \begin{bmatrix} N \\ k \end{bmatrix}_q$$

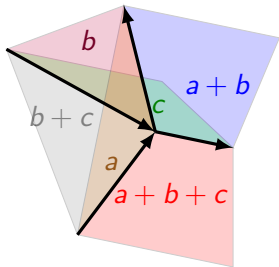
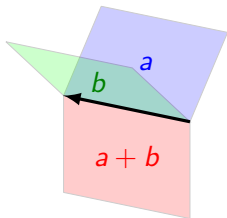
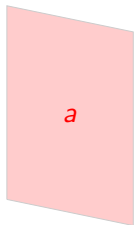
$$\langle \langle \begin{array}{c} m+n \uparrow \\ \downarrow m \end{array} \begin{array}{c} \leftarrow n \\ \rightarrow m \end{array} \rangle \rangle = \begin{bmatrix} N-m \\ n \end{bmatrix}_q \langle \langle \uparrow m \rangle \rangle$$

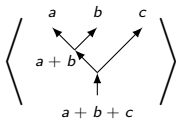
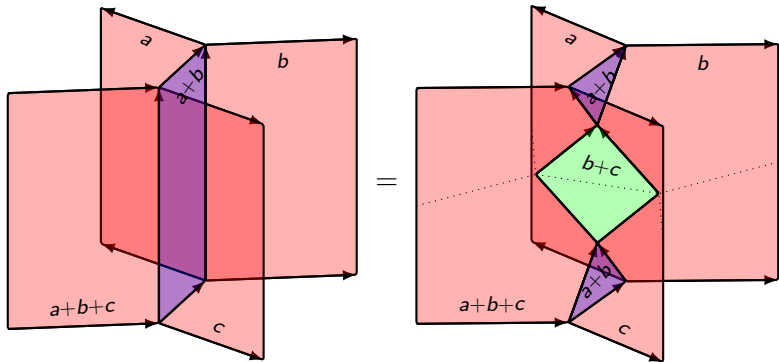
$$\langle \langle \begin{array}{c} i \quad j \quad k \\ \swarrow \quad \searrow \quad \nearrow \\ \uparrow \\ i+j+k \end{array} \rangle \rangle = \langle \langle \begin{array}{c} i \quad j \quad k \\ \swarrow \quad \searrow \quad \nearrow \\ \uparrow \\ i+j+k \end{array} \rangle \rangle$$

$$\langle \langle \begin{array}{c} m+n \uparrow \\ \downarrow m \end{array} \begin{array}{c} \leftarrow n \\ \rightarrow m \end{array} \rangle \rangle = \begin{bmatrix} m+n \\ m \end{bmatrix}_q \langle \langle \uparrow m+n \rangle \rangle$$

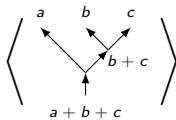
$$\langle \langle \begin{array}{c} 1 \quad m \\ \uparrow m+1 \quad \downarrow \\ \leftarrow \quad \rightarrow \\ \uparrow m+1 \quad \downarrow \\ 1 \quad m \end{array} \rangle \rangle = \langle \langle \uparrow 1 \rangle \rangle \langle \langle \downarrow m \rangle \rangle + [N-m-1]_q \langle \langle \begin{array}{c} 1 \quad m \\ \swarrow \quad \searrow \\ \uparrow \\ m-1 \end{array} \rangle \rangle$$

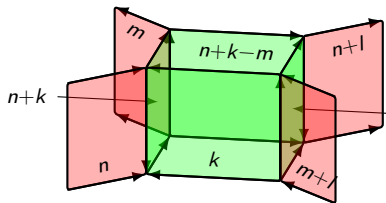
$$\langle \langle \begin{array}{c} m \quad n+l \\ \uparrow n+k \quad \downarrow \\ \leftarrow n+k-m \quad \rightarrow \\ \leftarrow k \quad \rightarrow \\ \uparrow n \quad \downarrow \\ m+l \end{array} \rangle \rangle = \sum_{j=\max(0, m-n)}^m \begin{bmatrix} l \\ k-j \end{bmatrix}_q \langle \langle \begin{array}{c} m \quad n+l \\ \uparrow m-j \quad \downarrow \\ \leftarrow j \quad \rightarrow \\ \leftarrow n+j-m \quad \rightarrow \\ \uparrow n \quad \downarrow \\ m+l \end{array} \rangle \rangle$$





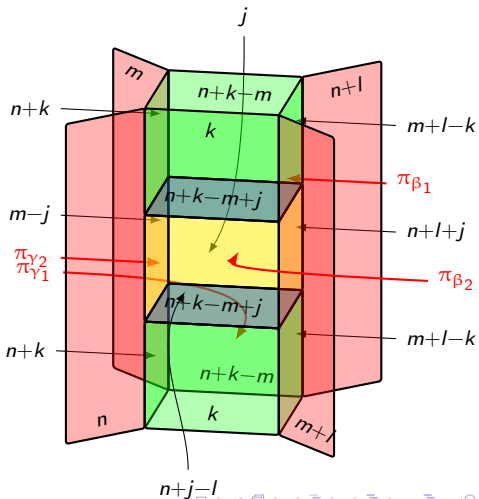
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$$m+l-k = \sum_{j=\max(0, m-n)}^m \sum_{\alpha \in T(k-j, l-k+j)}$$

$$(-1)^{|\alpha|+(l-k+j)(m-j)} \sum_{\substack{\beta_1, \beta_2 \\ \gamma_1, \gamma_2}} c_{\beta_1 \beta_2}^{\alpha} c_{\gamma_1 \gamma_2}^{\hat{\alpha}}$$







Thank you!