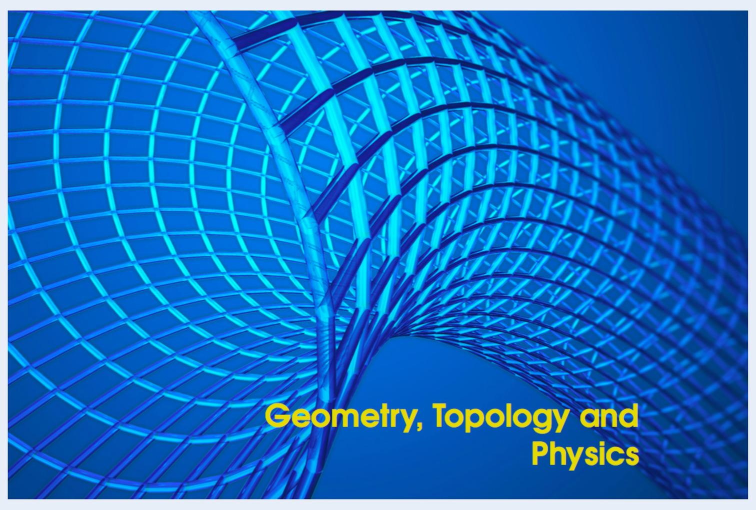


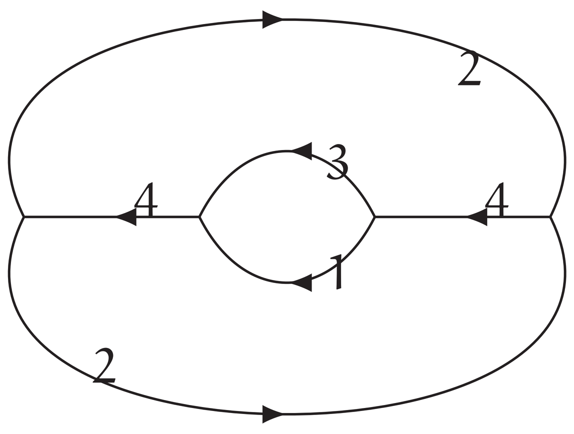
$\mathfrak{sl}_N$-EVALUATION OF FOAMS

LOUIS-HADRIEN ROBERT UNIVERSITÉ DE GENÈVE
JOINT WORK WITH EMMANUEL WAGNER UNIVERSITÉ DE BOURGOGNE

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

MOY GRAPHS [3]

A *MOY graph* is a planar, trivalent, oriented graph whose edges are labeled by positive integers and with conservative flow.

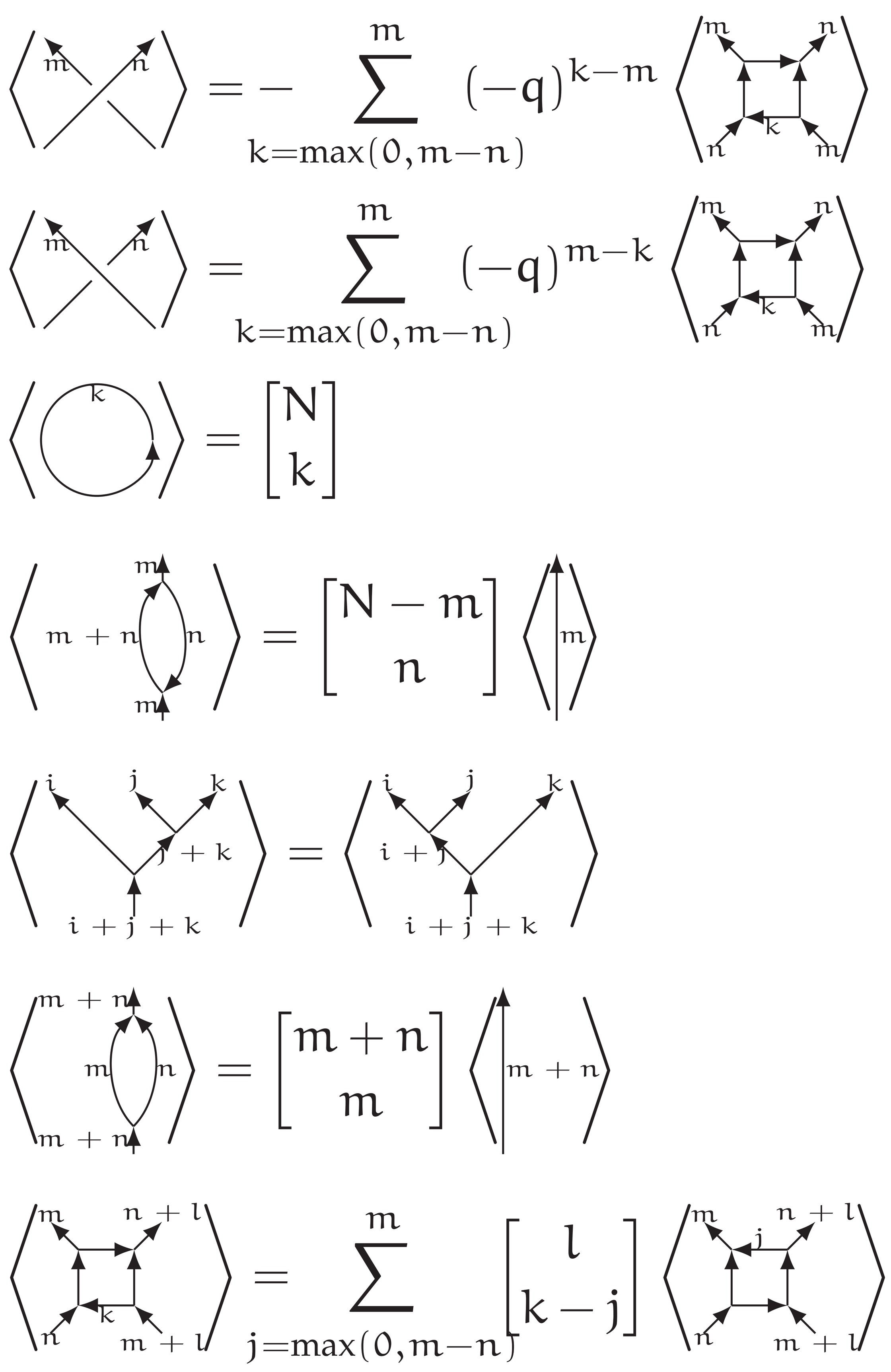


In the category Foam:

- Objects are MOY graphs,
- Morphisms are cobordisms between MOY graphs aka foams.

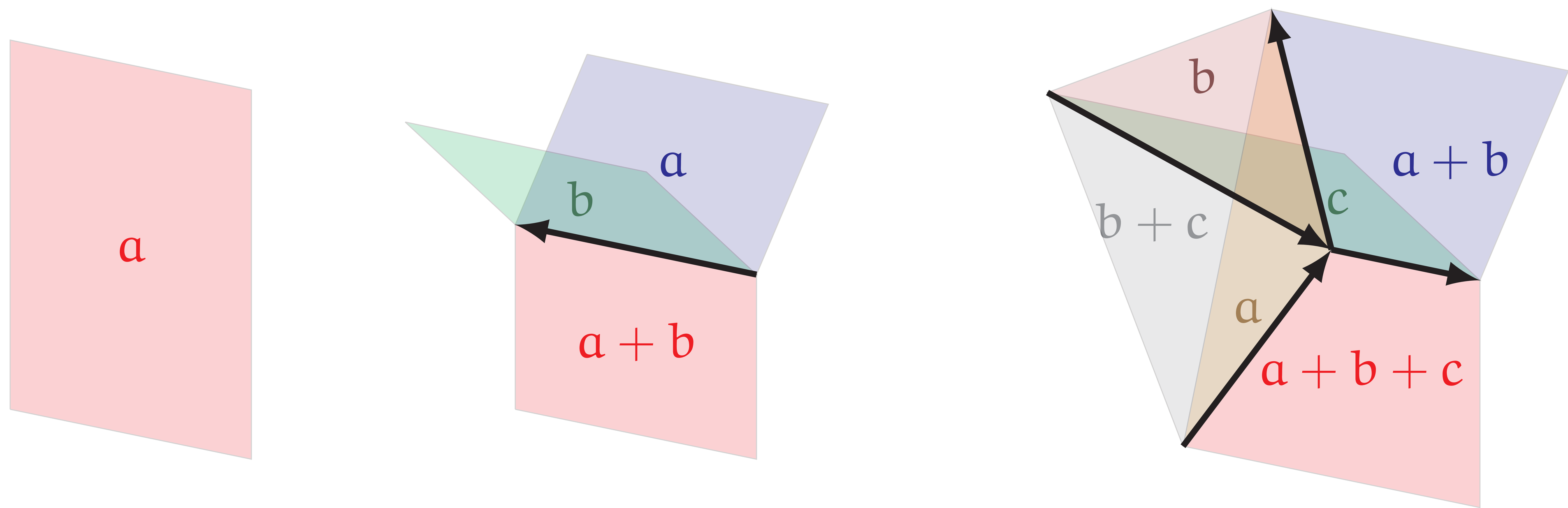
MOY CALCULUS [3]

Using the following local relations one can compute the Reshetikhin–Turaev link invariants associated with exterior powers of the standard representation of $U_q(\mathfrak{sl}_N)$:



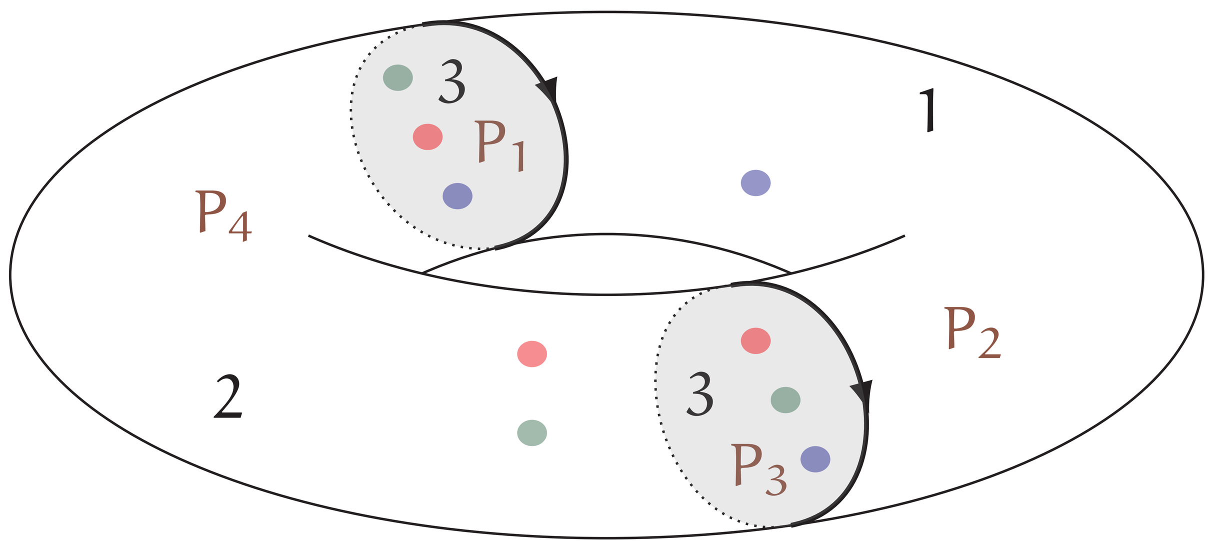
A MAGIC FORMULA: THE $\mathfrak{sl}_N$-FOAM EVALUATION

A *foam* is a finite CW-complex (with additional combinatorial data) with 3 local models:



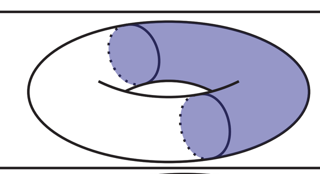
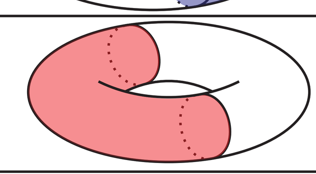
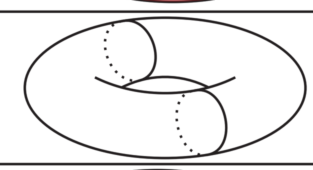
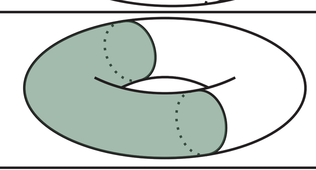
$$\tau_N(F) = \sum_{c \in \mathrm{col}_N(F)} \frac{(-1)^{\sum_{i=1}^{N} i\chi_i(F(c))/2 + \sum_{1\leq i<j\leq N} \theta^+_{ij}(F(c))} \prod_f P_f(c(f))}{\prod_{1\leq i<j\leq N} (y_i - y_j)^{\frac{\chi_{ij}(F(c))}{2}}} \in \mathbb{Z}[y_1, \dots, y_N]^{\mathfrak{S}_N}$$

EXAMPLE ($N = 4, \mathbb{P} = \{y_1, y_2, y_3, y_4\}$)

$F =$ 

$P_1 = t_1 + t_2 + t_3$
$P_2 = t_1^2$
$P_3 = 1$
$P_4 = t_1 t_2 (t_1 + t_2)$

	y_1y_2	y_1y_3	y_1y_4	y_2y_3	y_2y_4	y_3y_4
$\chi_{\bullet\bullet}$	0	2	0	2	0	2
$\theta^+_{\bullet\bullet}$	2	0	2	0	0	0

$\mathbb{P}$	Monochrome	$\chi_\bullet$
y_1		2
y_2		2
y_3		0
y_4		2

$$\frac{(-1)^{1+2+4+2+2}(y_1 + y_2 + y_4) y_1^2 (y_2 y_4 (y_2 + y_4))}{(y_1 - y_3)(y_2 - y_3)(y_3 - y_4)}$$

THEOREMS [4]

1. The $\mathfrak{sl}_N$-foam evaluation together with the universal construction [1] provides a functor

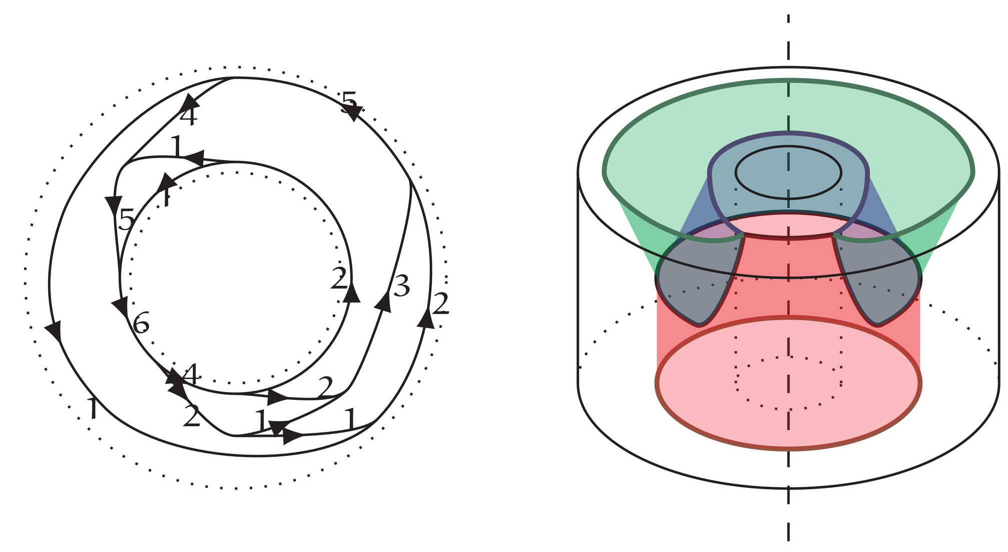
$$\mathcal{F}_N : \mathsf{Foam} \to \mathbb{Z}[y_1, \dots, y_N]\text{–}\mathrm{mod}_{gr}$$

which categorifies the MOY calculus.

2. The functor $\mathcal{F}_N$ together with the Rickard complexes associated with crossings provides an equivariant link homology which categorifies the Reshetikhin–Turaev invariants associated with exterior powers of the standard representation of $U_q(\mathfrak{sl}_N)$.

SYMMETRIC POWERS [5]

The same formula (modulo a level-rank duality) gives rise to similar theorems for *symmetric* powers of the standard representation of $U_q(\mathfrak{sl}_N)$. We need to change the category: we work with *vinyl graphs* and *tube-like foams*.



REFERENCES

[1] C. Blanchet, N. Habegger, G. Masbaum and P. Vogel. Topological quantum field theories derived from the Kauffman bracket. *Topology*, 34(4):883–927, 1995.

[2] P. Kronheimer and T. Mrowka. Tait colorings, and an instanton homology for webs and foams. http://arxiv.org/abs/1508.07205.

[3] H. Murakami, T. Ohtsuki, and S. Yamada. Homfly polynomial via an invariant of colored plane graphs. *Enseign. Math. (2)*, 44(3-4):325–360, 1998.

[4] L-.H. Robert and E. Wagner. A closed formula for the evaluation of $\mathfrak{sl}_n$-foams. https://arxiv.org/abs/1702.04140.

[5] L.-H. Robert and E. Wagner. Symmetric Khovanov–Rozansky link homologies. https://arxiv.org/abs/1801.02244.

PROJECTS

- Adapt the formula to deal with instanton homology of webs [2] (with M. Khovanov).
- Modify the symmetric theory at $N = 1$ in order to categorify the Alexander polynomial (with E. Wagner).
- Find an analogue formula for foams of type D (with E. Wagner).
- Give a foamy interpretation of the Hochschild homology of Soergel bimodules.