Foams and categorification

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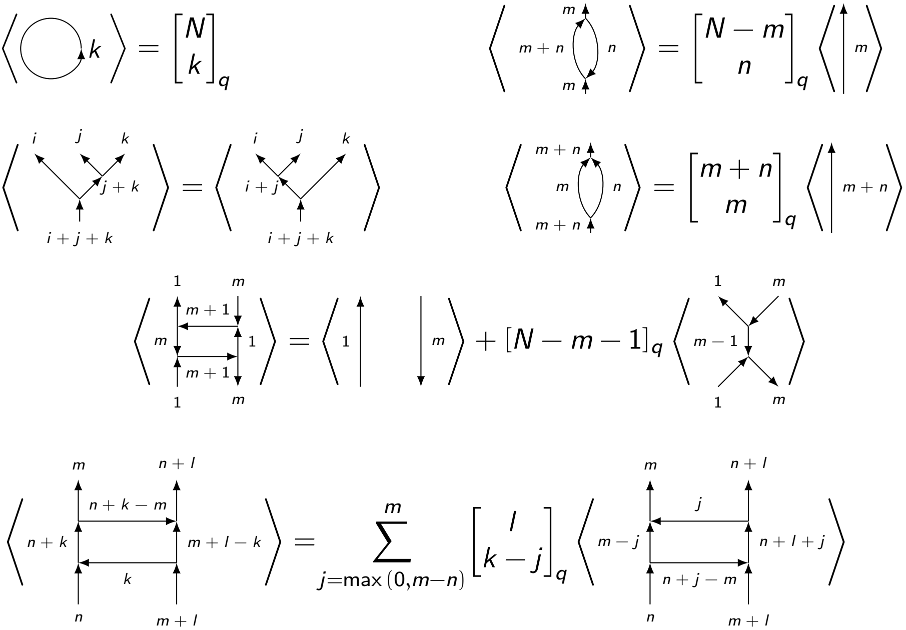
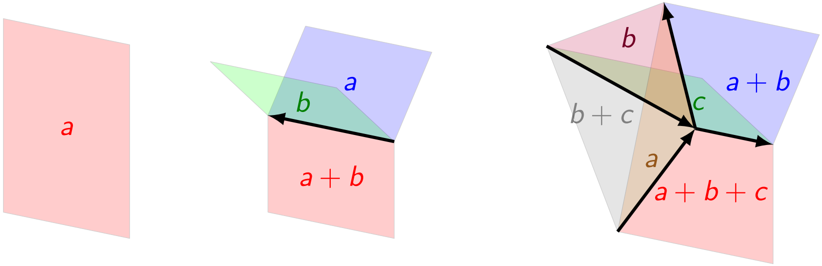
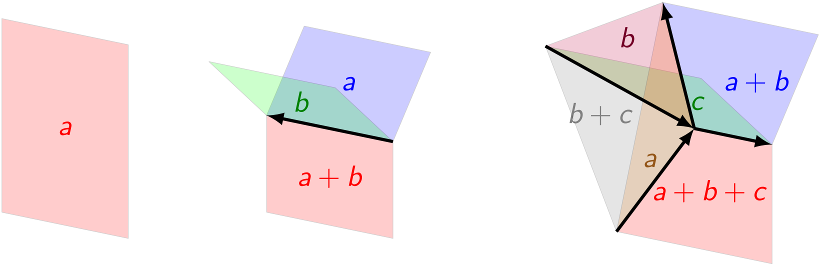
Workshop Categorification in Mathematical Physics

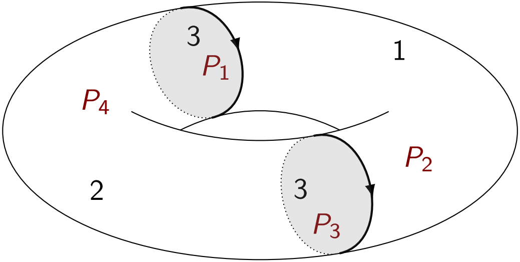

k
= [N k]_q
m
m + n
n
m
= [N − m n]_q
m
i j k
j + k
i + j + k
=
i j k
i + j
i + j + k
m + n
m
n
m + n
= [m + n m]_q
m + n
1 m
m + 1
m
1
m + 1
1 m
= 1 m
+ [N − m − 1]_q
1 m
m − 1
1 m
m n + l
n + k − m
n + k
m + l − k
k
n m + l
= Σ_{j=max(0,m−n)}^{m} [l k − j]_q
m n + l
j
m − j
n + l + j
n + j − m
n m + l


a
a
b
a + b
b
a + b
c
b + c
a
a + b + c



Definition (R.–Wagner, ’17)

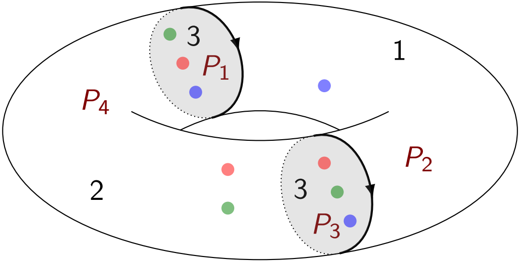
$$\tau_N(F) = \sum_c \frac{(-1)^{\sum_{i=1}^N i\chi_i(F(c))/2 + \sum_{1\le i<j\le N} \theta_{ij}^+(F(c))} \prod_f P_f(c(f))}{\prod_{1\le i<j\le N}(X_i - X_j)^{\frac{\chi_{ij}(F(c))}{2}}}$$



$P_1 = t_1 + t_2 + t_3$ $\qquad P_2 = t_1^2$

$P_3 = 1$ $\qquad P_4 = t_1 t_2 (t_1 + t_2)$

$$N = 4$$
$$\mathbb{P} = \{X_1, X_2, X_3, X_4\}.$$



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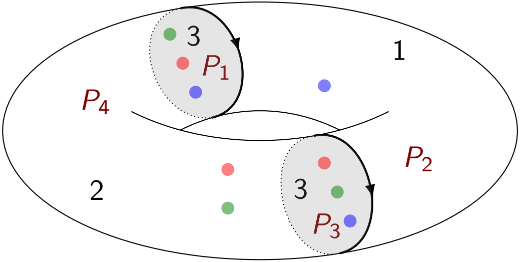

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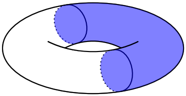
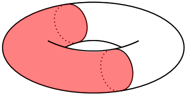
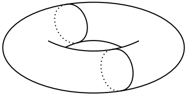
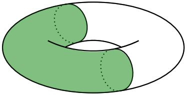
$\mathbb{P}$	Monochrome	$\chi_\bullet$
X_1		2
X_2		2
X_3		0
X_4		2

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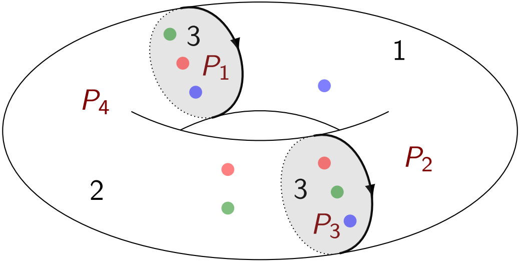

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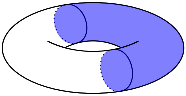
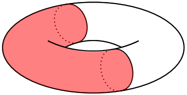
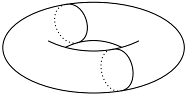
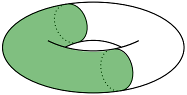
$\mathbb{P}$	Monochrome	$\chi_\bullet$
X_1		2
X_2		2
X_3		0
X_4		2

	X_1X_2	X_1X_3	X_1X_4	X_2X_3	X_2X_4	X_3X_4
$\chi_{\bullet\bullet}$	0	2	0	2	0	2
$\theta^+_{\bullet\bullet}$	2	0	2	0	0	0

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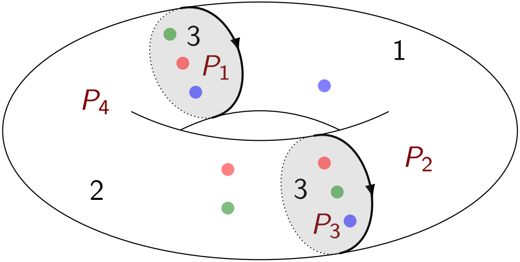
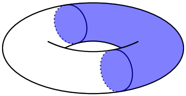
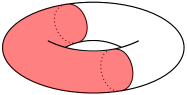
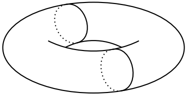
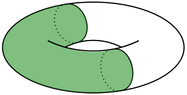


$P_1 = t_1 + t_2 + t_3 \qquad P_2 = t_1^2$

$P_3 = 1 \qquad P_4 = t_1 t_2 (t_1 + t_2)$

$$\frac{(-1)^{1+2+4}(X_1 + X_2 + X_4)X_1{}^2(X_2X_4(X_2 + X_4))}{(X_1 - X_3)(X_2 - X_3)(X_3 - X_4)}$$

$\mathbb{P}$	Monochrome	$\chi_\bullet$
X_1		2
X_2		2
X_3		0
X_4		2

	X_1X_2	X_1X_3	X_1X_4	X_2X_3	X_2X_4	X_3X_4
$\chi_{\bullet\bullet}$	0	2	0	2	0	2
$\theta^+_{\bullet\bullet}$	2	0	2	0	0	0

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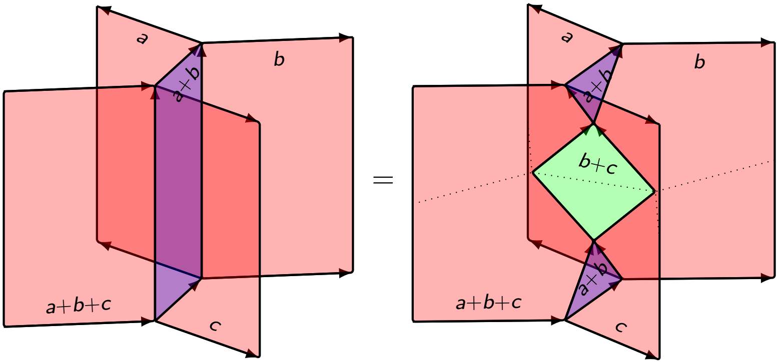
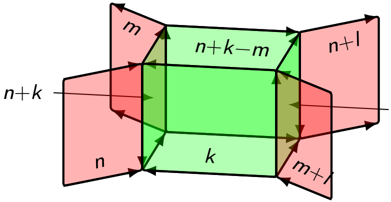
$$\mathbb{P} = \{X_1, X_2, X_3, X_4\}.$$

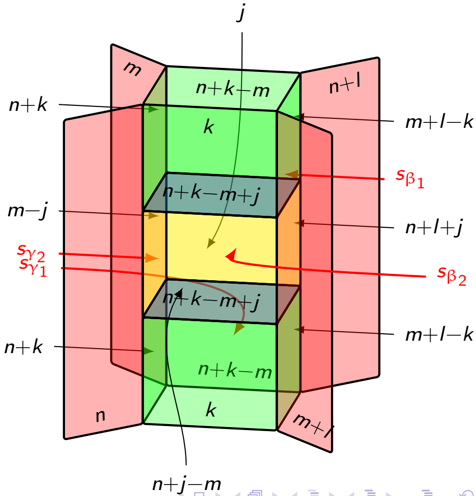


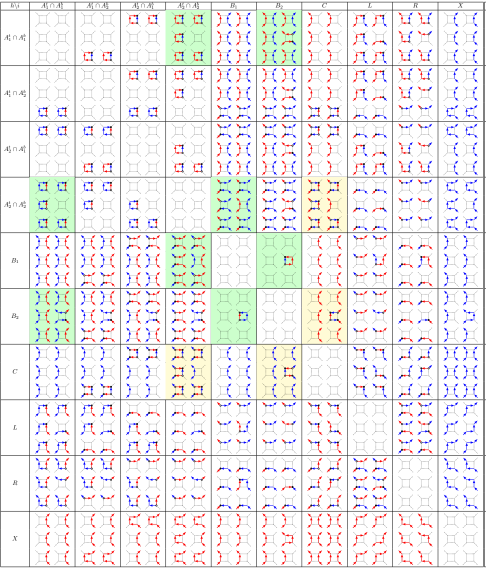
a
b
a+b
a+b+c
c
=
a
b
a+b
b+c
a+b
a+b+c
c

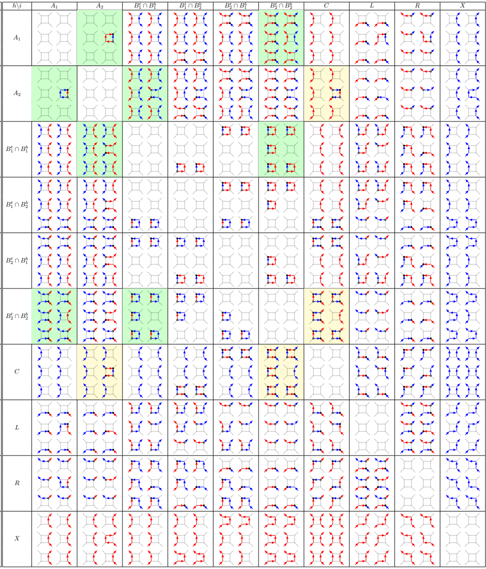


$$= \sum_{j=\max(0,m-n)}^{m} \; \sum_{\alpha \in T(k-j,\, l-k+j)}$$

$$(-1)^{|\alpha|+(l-k+j)(m-j)} \sum_{\substack{\beta_1,\beta_2\\ \gamma_1,\gamma_2}} c^{\alpha}_{\beta_1\beta_2}\, c^{\widehat{\alpha}}_{\gamma_1\gamma_2}$$




$h\backslash i$
$A_1^t \cap A_1^b$
$A_1^t \cap A_2^b$
$A_2^t \cap A_1^b$
$A_2^t \cap A_2^b$
B_1
B_2
C
L
R
X


$h\backslash i$
A_1
A_2
$B_1^t \cap B_1^b$
$B_1^t \cap B_2^b$
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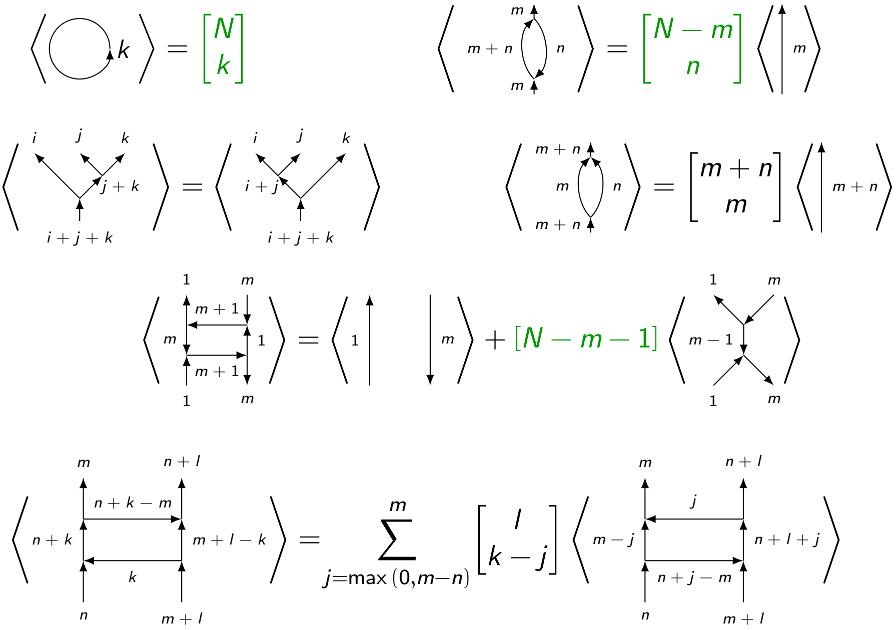
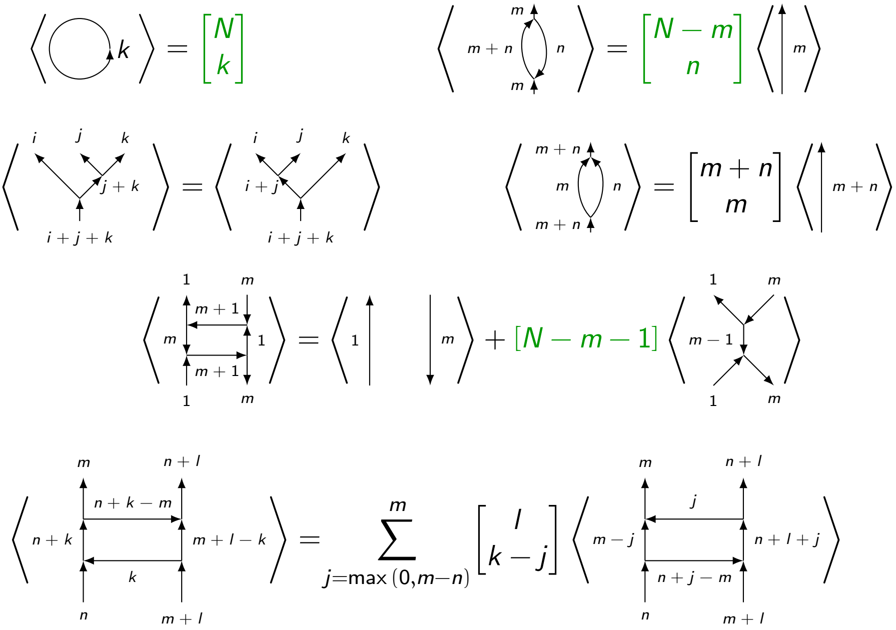
C
L
R
X


k
= [N k]
m
m + n
n
m
= [N − m n]
m
i j k
j + k
i + j + k
=
i j k
i + j
i + j + k
m + n
m
n
m + n
= [m + n m]
m + n
1 m
m + 1
m
1
m + 1
1 m
= 1 m
+ [N − m − 1]
1 m
m − 1
1 m
m n + l
n + k − m
n + k
m + l − k
k
n m + l
= $\sum_{j=\max(0,m-n)}^{m}$ [l k − j]
m n + l
j
m − j
n + l + j
n + j − m
n m + l

From $\Lambda^\bullet$ to $\mathrm{Sym}^\bullet$: N goes to $-N$.



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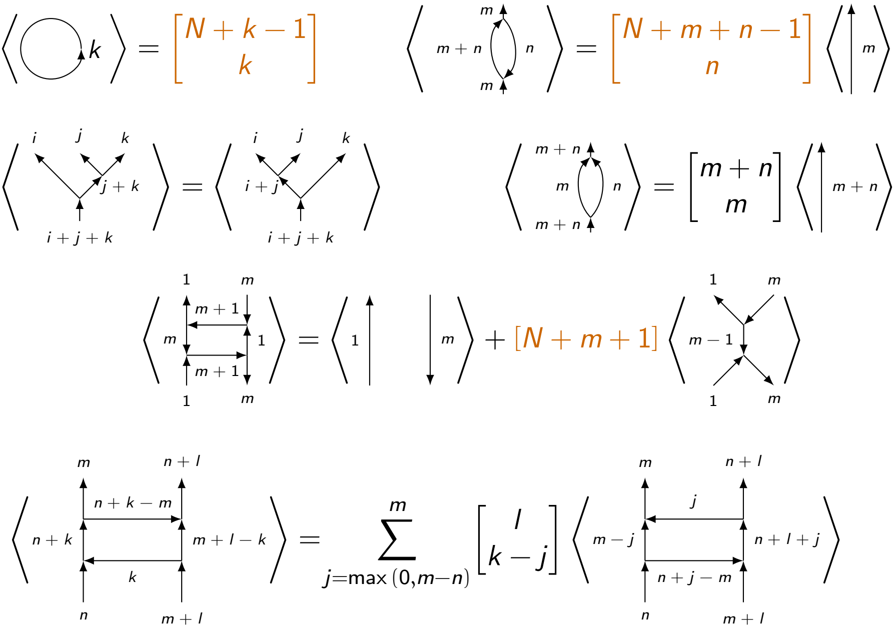

Thank you !!