Categorification of the symmetric $U_q(\mathfrak{sl}_N)$ link invariant

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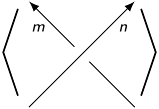
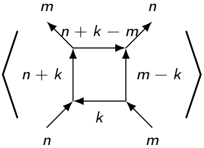
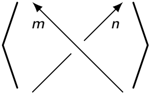
http://www.unige.ch/math/folks/robert/wb8.pdf

The $\mathfrak{sl}_N$-link invariant



$$= \sum_{k=\max(0,m-n)}^{m} (-1)^{m-k} q^{k-m}$$





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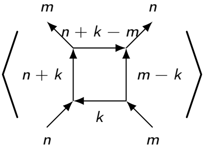
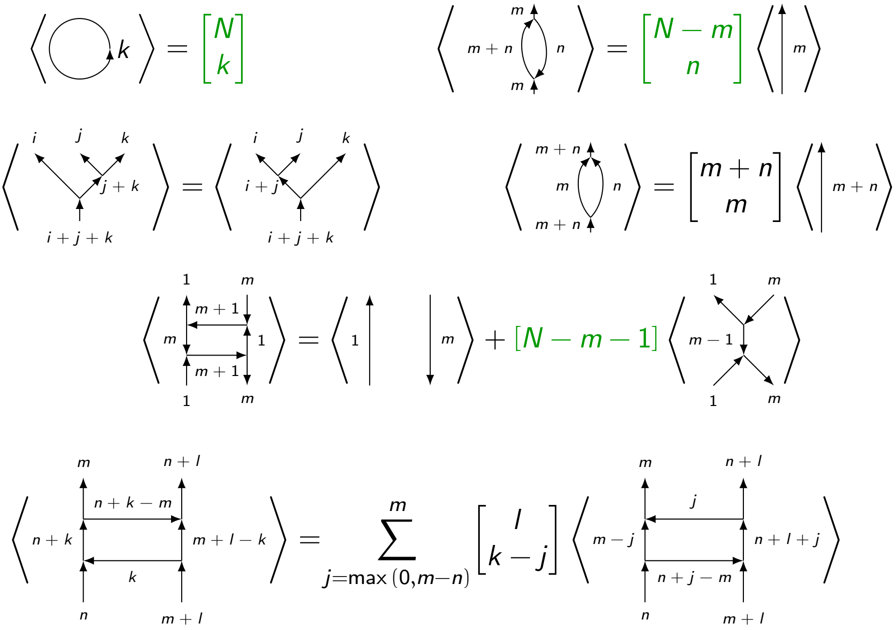
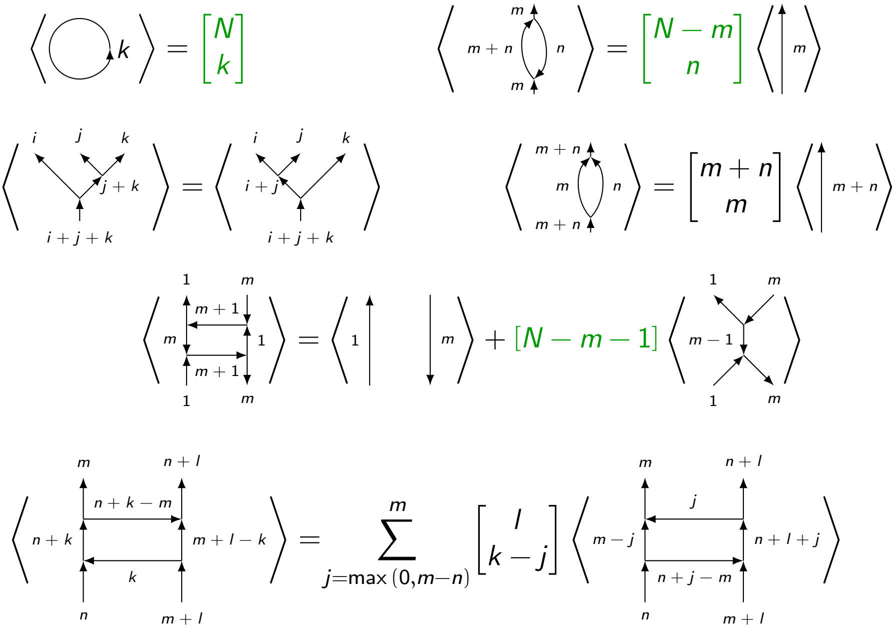



k
= [N k]
m
m + n
n
m
= [N − m n]
m
i
j
k
j + k
i + j + k
=
i
j
k
i + j
i + j + k
m + n
m
n
m + n
= [m + n m]
m + n
1
m
m + 1
m
1
m + 1
1
m
= 1
m
+ [N − m − 1]
1
m
m − 1
1
m
m
n + l
n + k − m
n + k
m + l − k
k
n
m + l
=
$\sum_{j=\max(0,m-n)}^{m}$
[l k − j]
m
n + l
j
m − j
n + l + j
n + j − m
n
m + l

From $\Lambda^\bullet$ to $\mathrm{Sym}^\bullet$: N goes to $-N$.



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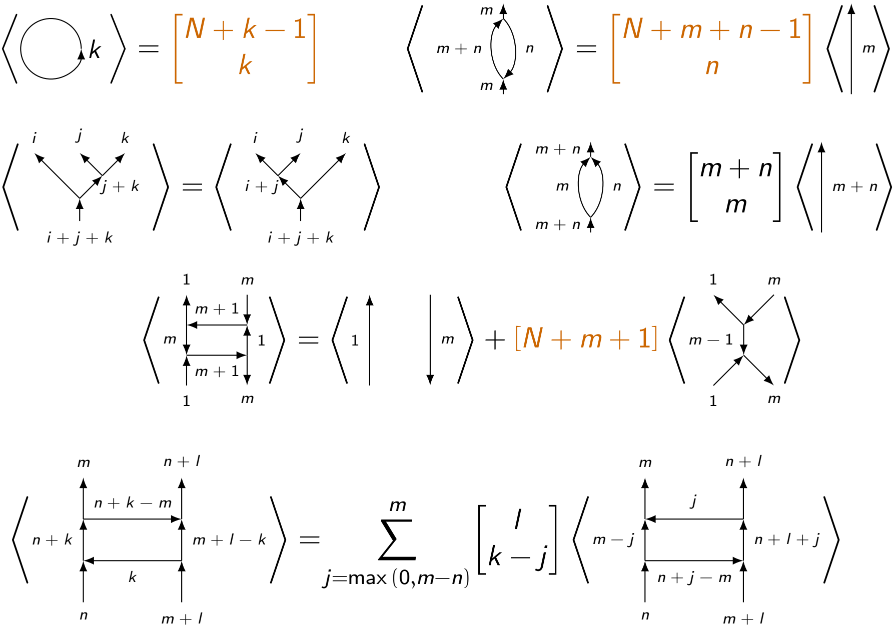
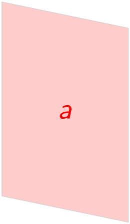
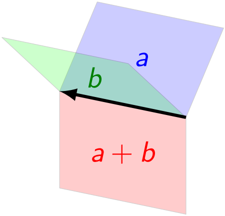
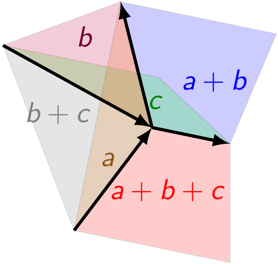
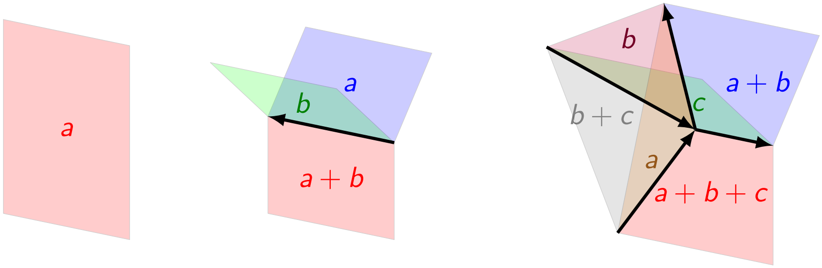



a


a
b
$a+b$


b
$a+b$
c
$b+c$
a
$a+b+c$



Theorem (Categorification of the $\Lambda^\bullet$-MOY calculus)

There exists a foamy TQFT $\mathcal{F}_N \colon \mathrm{Foam} \to R_N\text{-mod}_{\mathrm{gr}}$ *such that*

$$\dim_q \mathcal{F}_N(\Gamma) = \langle \Gamma \rangle \,.$$

It can be extended to an homological link invariant.

Bad news

It is not possible to categorify the $\mathrm{Sym}^\bullet$-MOY calculus with such a TQFT.

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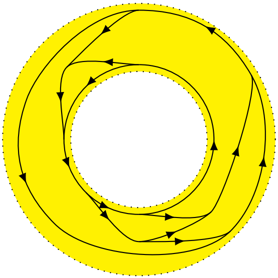
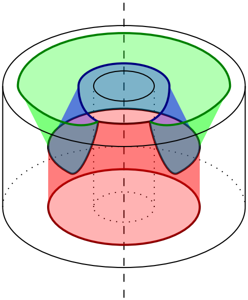
We restrict the class of graphs and the class of foams:



Vinyl graphs



Tube-like foams

Theorem (R.–Wagner, '18)

There exists a foamy restricted TQFT $\mathcal{F}_N \colon \mathsf{TLFoam} \to R_N\text{-mod}_{\mathrm{gr}}$ *such that*

$$\dim_q \mathcal{F}_N(\Gamma) = \langle \Gamma \rangle \,.$$

It can be extended to an homological link invariant.

Thank you!

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