$\mathfrak{gl}_0$-knot homology I

Louis-Hadrien Robert

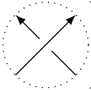
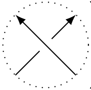
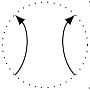
Emmanuel Wagner

UNIVERSITÉ DE GENÈVE

UNIVERSITÉ DE BOURGOGNE

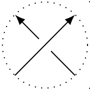
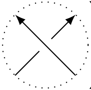
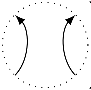
Zurich

- The $\mathfrak{gl}_1$ link invariant P_1.

$qP_1\Big($  $\Big) - q^{-1}P_1\Big($  $\Big) = (q - q^{-1})P_1\Big($  $\Big)$

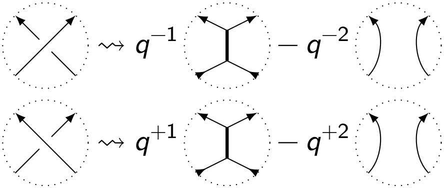
$L \mapsto 1 \in \mathbb{Z}[q, q^{-1}]$

- The Alexander polynomial Δ.

$\Delta\Big($  $\Big) - \Delta\Big($  $\Big) = (q - q^{-1})\Delta\Big($  $\Big)$

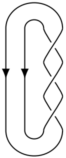
$\mathfrak{gl}_1$ invariant

Link diagram $\rightsquigarrow \mathbb{Z}[q, q^{-1}]$-lin. comb. of plane graphs

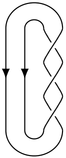


plane graph $\rightsquigarrow$ element of $\mathbb{N}[q, q^{-1}]$

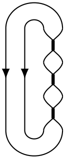
$\mathfrak{gl}_1$ invariant – Example



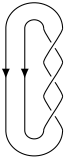
$\mathfrak{gl}_1$ invariant – Example



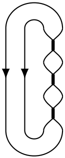
q^{-3}



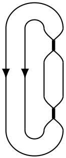
$\mathfrak{gl}_1$ invariant – Example



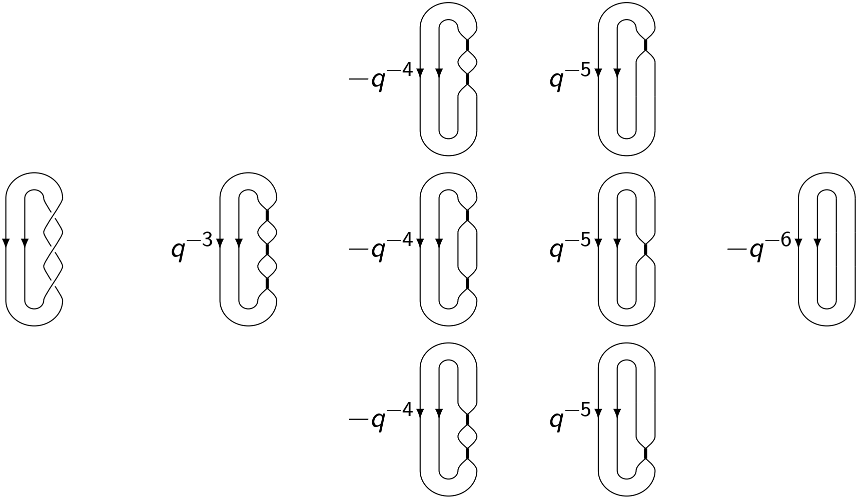
q^{-3}



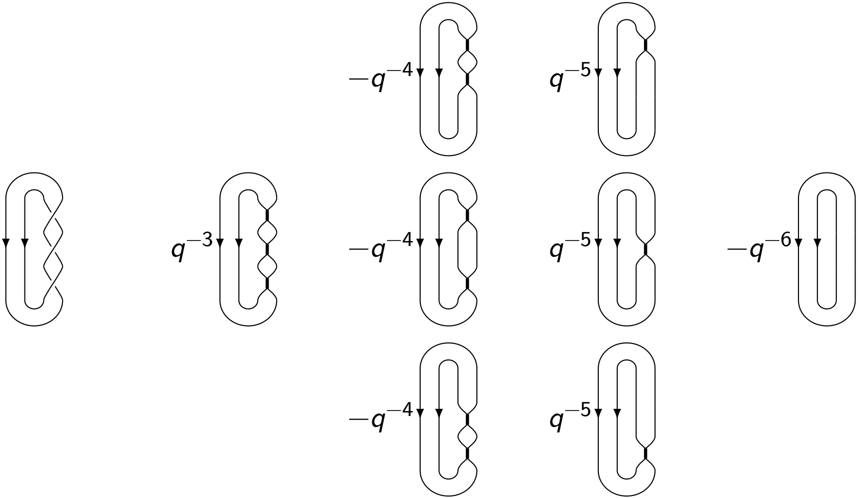
$-q^{-4}$



$\mathfrak{gl}_1$ invariant – Example



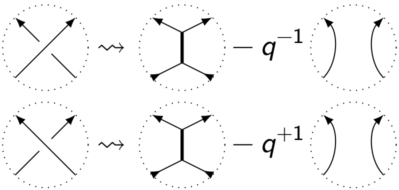
$\mathfrak{gl}_1$ invariant – Example



$$1 = q^{-3}[2]^3 \quad -3q^{-4}[2]^2 \quad +3q^{-5}[2] \quad -q^{-6}$$

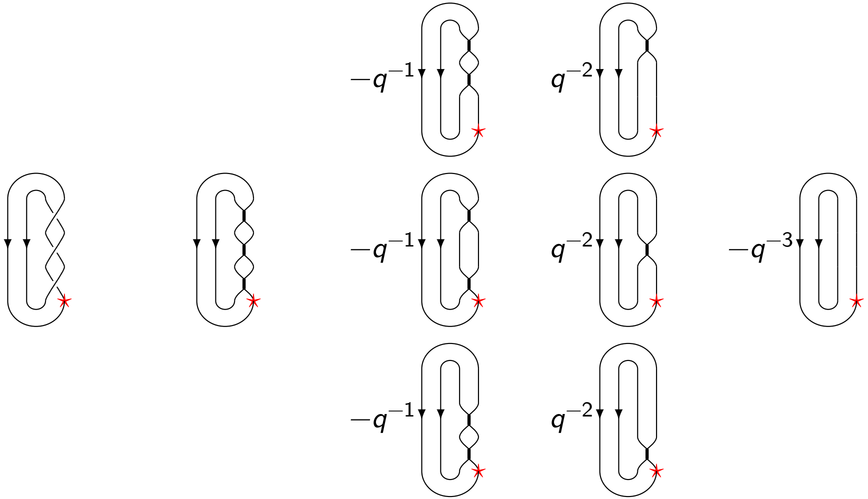
Alexander polynomial

Marked ($\star$) braid closure $\rightsquigarrow \mathbb{Z}[q, q^{-1}]$-lin. comb. of marked plane graphs

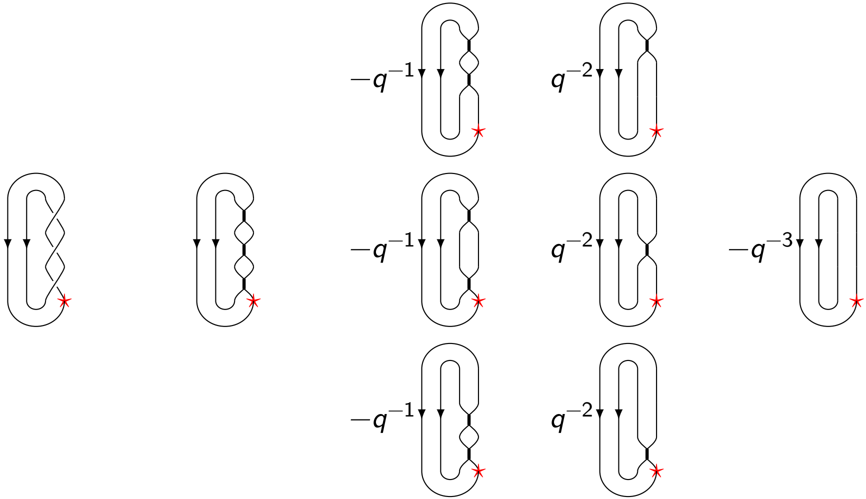


Marked plane graph $\rightsquigarrow$ element of $\mathbb{N}[q, q^{-1}]$

Alexander polynomial – Example



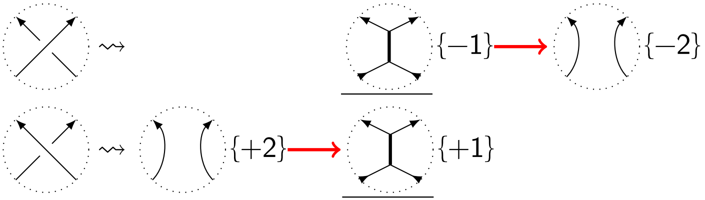
Alexander polynomial – Example



$$q^2 - 1 + q^{-2} \; = \qquad [2]^2 \qquad -3q^{-1}[2] \qquad +3q^{-2} \qquad 0$$

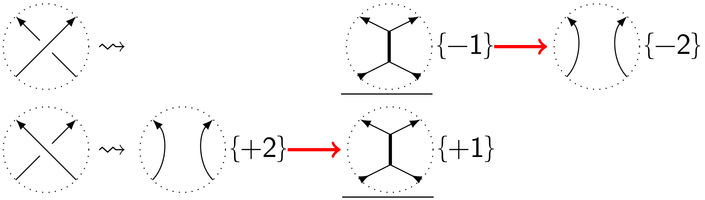
$\mathfrak{gl}_1$-homology

Braid closure diagram $\rightsquigarrow$ hypercube of plane graphs graphs (with shifts)



$\mathfrak{gl}_1$-homology

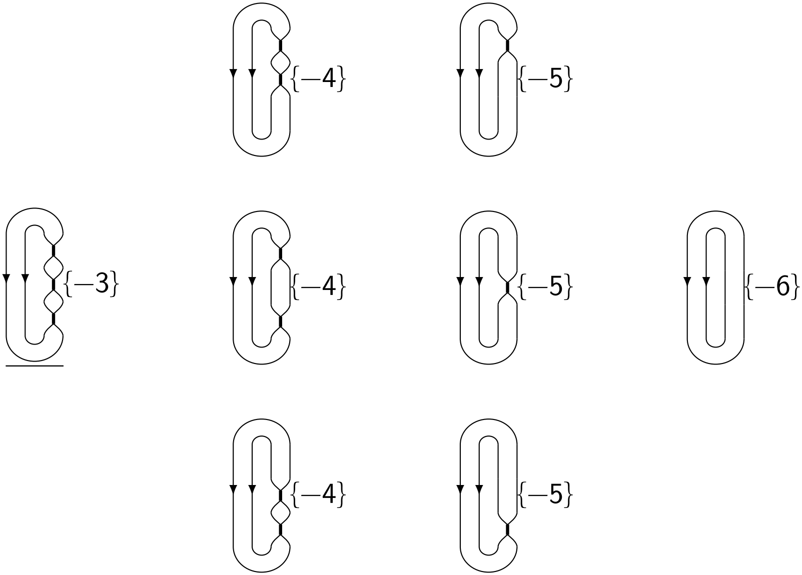
Braid closure diagram $\rightsquigarrow$ hypercube of plane graphs graphs (with shifts)



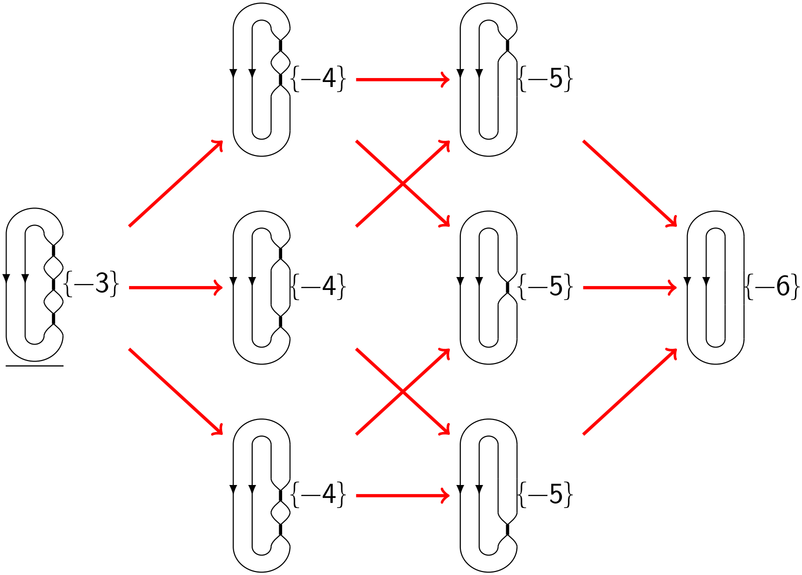
$\mathcal{F}_1$: Planar (vinyl) graph $\rightsquigarrow$graded vector space

$\longrightarrow$ $\rightsquigarrow$graded linear map

$\mathfrak{gl}_1$ homology – Example

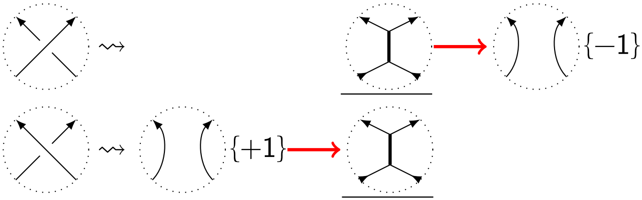


$\mathfrak{gl}_1$ homology – Example



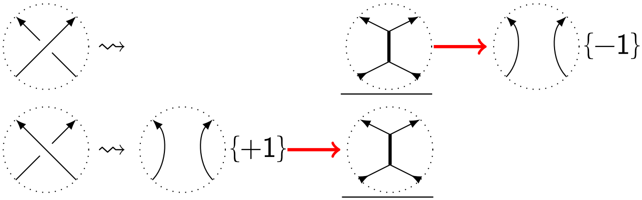
$\mathfrak{gl}_0$-homology

Marked ($\star$) braid closure $\rightsquigarrow$ hypercube of marked plane graphs (with shifts)



$\mathfrak{gl}_0$-homology

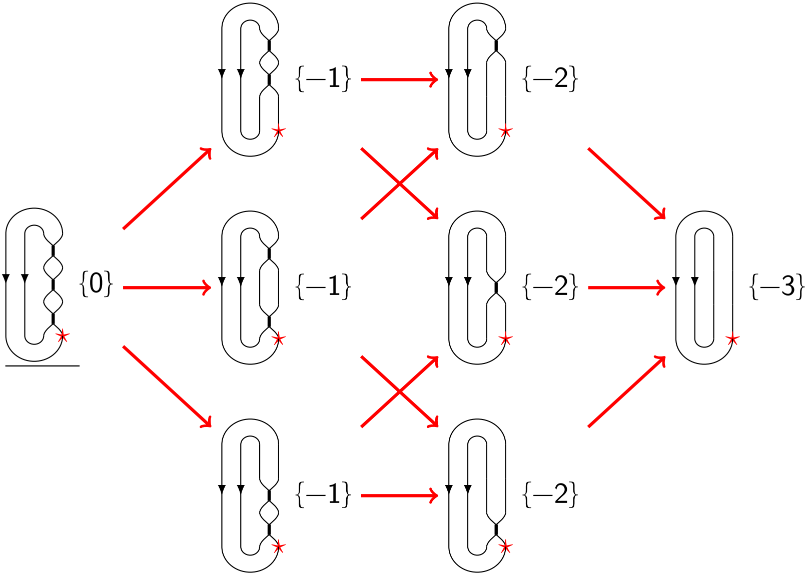
Marked ($\star$) braid closure $\rightsquigarrow$ hypercube of marked plane graphs (with shifts)



$\mathcal{F}_0'$: Marked vinyl graph $\rightsquigarrow$graded vector space

$\longrightarrow$ $\rightsquigarrow$graded linear map

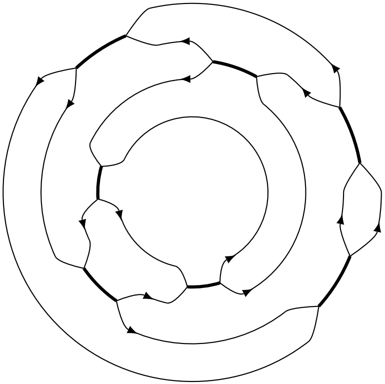
$\mathfrak{gl}_0$ homology – Example



Today's aims

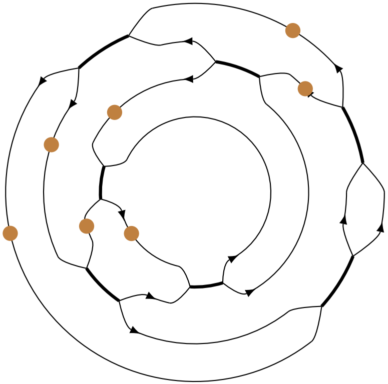
- Annular combinatorics,
- Define functors $\mathcal{F}_1$ and $\mathcal{F}_0'$.

Vinyl graph $\rightsquigarrow$ vector space



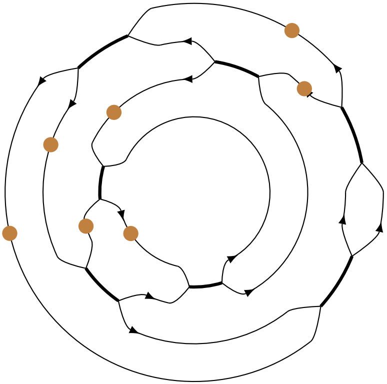
Vinyl graph $\Gamma \circlearrowleft$ index k.

Vinyl graph ⇝ vector space



Vinyl graph Γ ↻ index k.
Dot configuration d ●.
$\deg(d) = 2\#$● $- \#V(\Gamma)/2$

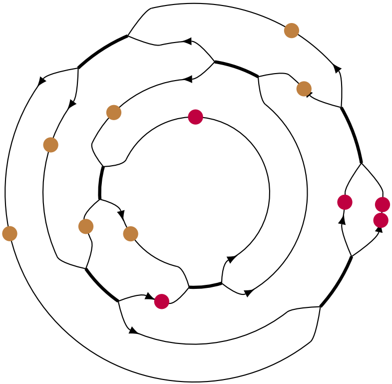
Vinyl graph $\rightsquigarrow$ vector space



Vinyl graph $\Gamma \circlearrowleft$ index k.
Dot configuration d ●.
$\deg(d) = 2\#● - \#V(\Gamma)/2$

$$D(\Gamma) = \bigoplus_{d} \mathbb{Q}.$$

Vinyl graph $\rightsquigarrow$ vector space



Vinyl graph $\Gamma \circlearrowleft$ index k.
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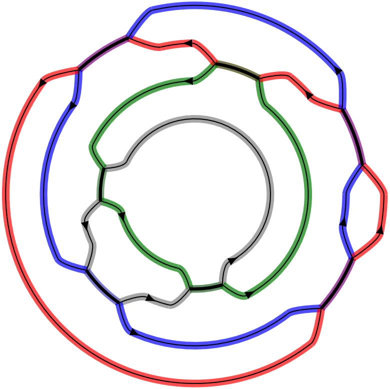
$\deg(d) = 2\#● - \#V(\Gamma)/2$

$$D(\Gamma) = \bigoplus_{d} \mathbb{Q}.$$

Multiplication μ on $D(\Gamma)$.

Vinyl graph ⇝ vector space



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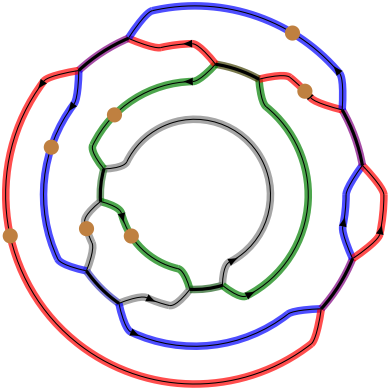
$$D(\Gamma) = \bigoplus_{d} \mathbb{Q}.$$

Multiplication μ on $D(\Gamma)$.
Coloring $c = (C_1, C_2, \ldots, C_k)$
$\Gamma = \bigsqcup_i C_i$.

Vinyl graph $\rightsquigarrow$ vector space



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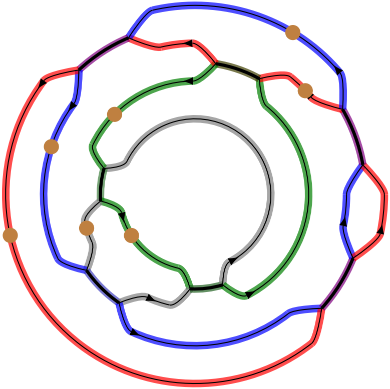
$$\tau(d,c) = \frac{\prod_{i=1}^{k} X_i^{\#\{\bullet \text{ in } C_i\}}}{\prod (X_i - X_j)}$$

Vinyl graph ⇝ vector space



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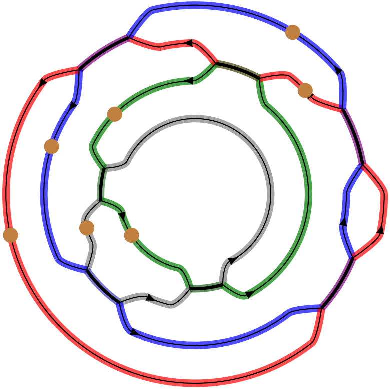
$$\tau(d,c) = \frac{\prod_{i=1}^{k} X_i^{\#\{\bullet \text{ in } C_i\}}}{\prod (X_i - X_j)} = \frac{-X_1{}^2 X_2{}^2 X_3 X_4{}^2}{(X_1 - X_2)^3 (X_3 - X_4)^2 (X_1 - X_4)(X_2 - X_3)}$$

C_i C_j

Vinyl graph $\rightsquigarrow$ vector space



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Multiplication μ on $D(\Gamma)$.
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$$\tau(d,c) = \frac{\prod_{i=1}^{k} X_i^{\#\{● \text{ in } C_i\}}}{\prod (X_i - X_j)}$$

$$\tau(d) = \sum_{c \in \mathrm{col}(\Gamma)} \tau(d,c)$$