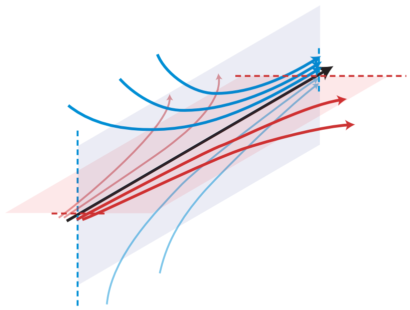
Question 1 (Fried 1982)**.** *Does every transitive Anosov flow admit a finite collection of periodic orbits that form a genus-one fibered link?*

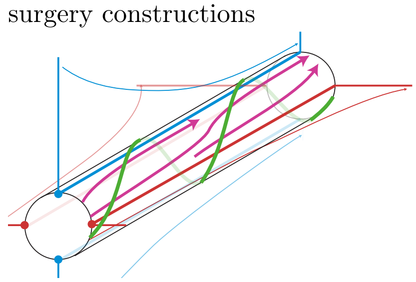
Question 2 (Ghys 80's)**.** *Given any two transitive Anosov flows, do they differ by a finite number of Dehn surgeries along periodic orbits?*

(1) Anosov flows



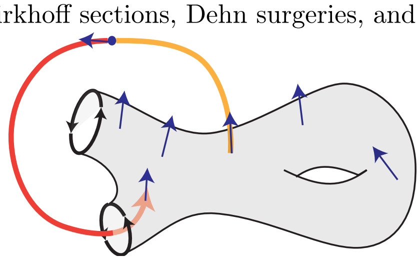
examples:

(a) suspension of a hyperbolic matrix $A \in \mathrm{SL}_2(\mathbb{Z})$
(b) geodesic flow in negative curvature
(c) surgery constructions



Ratner (1968): existence of Markov partitions

(2) Birkhoff sections, Dehn surgeries, and almost equivalence



Theorem 3 (Fried 1982)**.** *Every transitive Anosov flow admits a Birkhoff section.*

Question 4 (Fried)**.** *(=question 1) Does every transitive Anosov flow admit a genus-one Birkhoff section?*

Question 5 (Ghys)**.** *(=question 2) Are all transitive Anosov flows pairwise almost-equivalent?*

(3) Equivalence of questions 1, 2 (Minakawa, unpublished)

Theorem 6 (Minakawa 2013)**.** $\forall A \in \mathrm{SL}_2(\mathbb{Z}), \mathrm{tr}(A) \geq 3$, $(M_A, \varphi_{\mathrm{sus}}) \overset{a.e.}{\longleftrightarrow} (M_{\left(\begin{smallmatrix}2&1\\1&1\end{smallmatrix}\right)}, \varphi_{\mathrm{sus}})$.

Corollary 7. *All Anosov flows admitting a genus-one Birkhoff section are almost-equivalent.*

(4) Positive answer for algebraic An. flows (D-Shannon, 2019)

Theorem 8 (Dehornoy-Shannon 2019)**.** *For every hyperbolic 2-orbifold* Σ, $(T^1\Sigma, \varphi_{\mathrm{geod}})$ *admits a genus-one Birkhoff section.*

(5) Toward a positive answer for all Anosov flows (D, work in progress)

Theorem 9 (Dehornoy 2020)**.** *If a transitive Anosov flow admits a* **genus-two** *Birkhoff section, then it admits a* **genus-one** *Birkhoff section.*