Plan of the talk

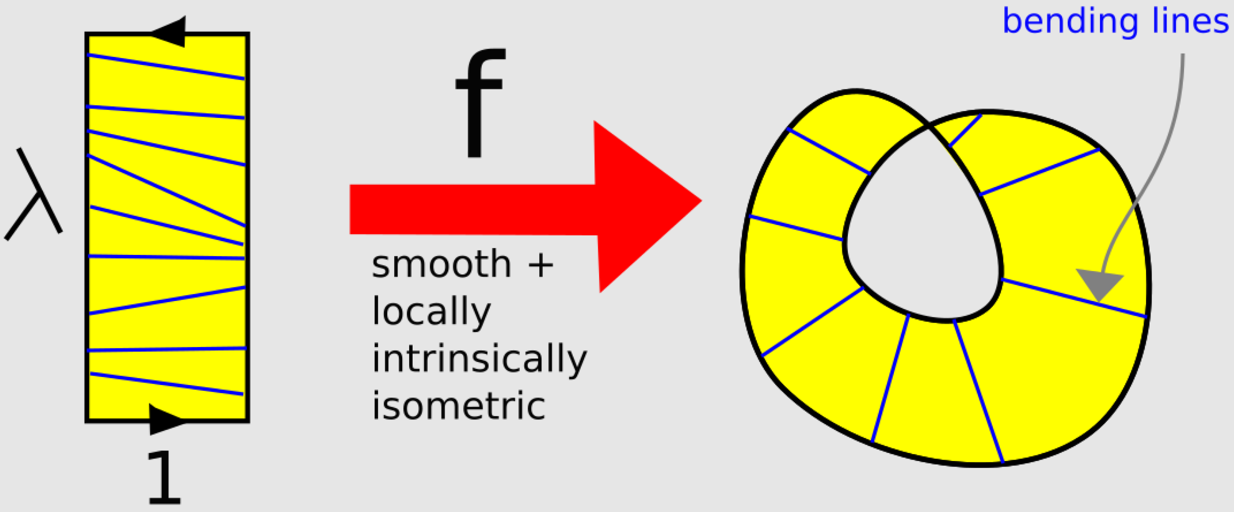
1. Definition of paper Moebius bands

2. Explanation of previous bounds.

3. Sketch of my improved bound.

4. My reduction of the main conjectures to finite dimensional "energy minimization" calculations.

Smooth paper Moebius band:

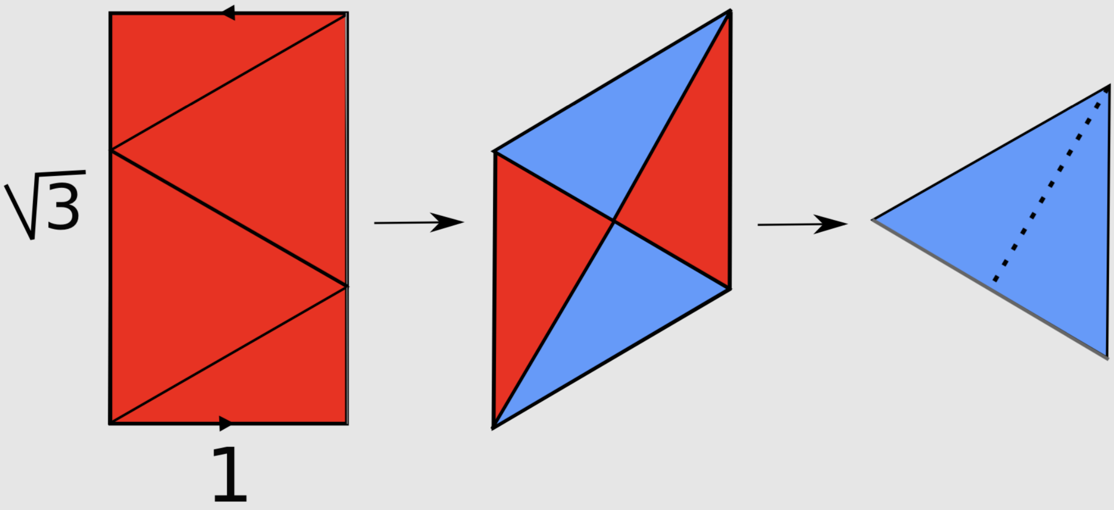


Q: (B. Halpern and C. Weaver 1977):

How small can λ be if f is an embedding?

I read about this in the book
Mathematical Omnibus (Ch 14)
by Dmitry Fuchs and Sergei Tabachnikov.

Example:

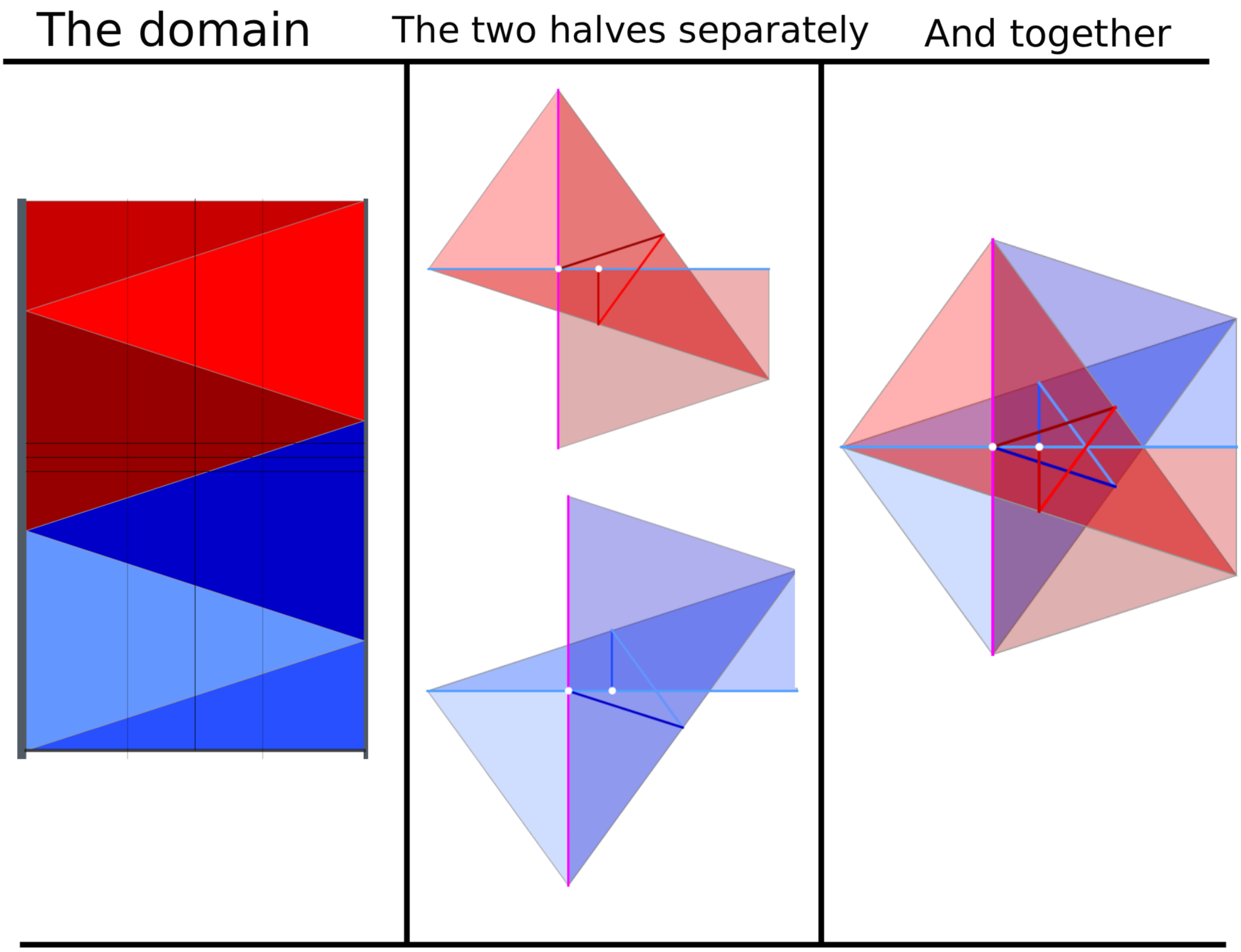


This example is not smooth but it is smoothly approximable. So any

$$\sqrt{3} + \varepsilon$$

is possible.

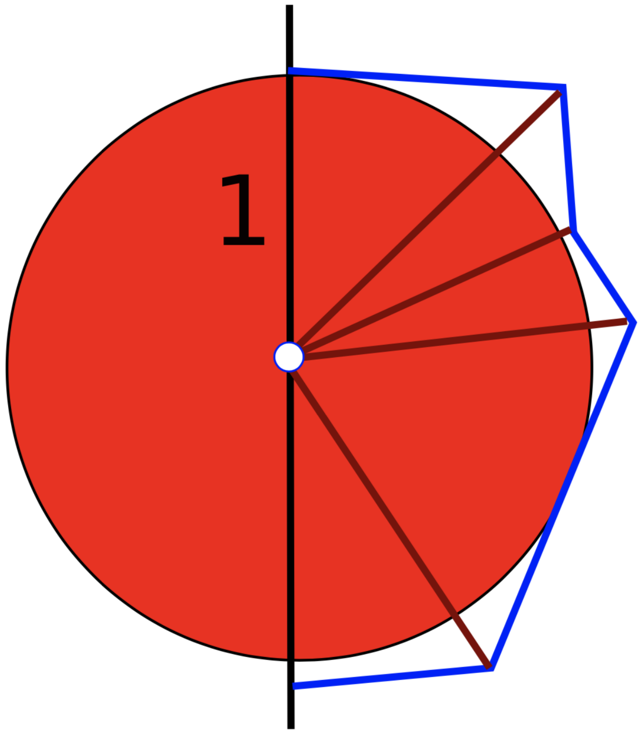
Example:



This is part of a series of examples of IMMERSED examples whose aspect ratio tends to Pi/2

Theorem [HW]:
Minimum lies in [Pi/2, $\sqrt{3}$]

Proof: 1. Example gives upper bound.
2, Ridge curve gives lower bound:



Note: the lower bound proof I learned
in [FT] is different, but of course related.

Theorem 1: Infimal aspect ratio is at least

$$\lambda_1 = \frac{2\sqrt{4-2\sqrt{3}}+4}{\sqrt[4]{3}\sqrt{2}+2\sqrt{2\sqrt{3}-3}} = 1.69497...$$

For comparison

Pi/2=1.508...

$\sqrt{3}$ = 1.732...

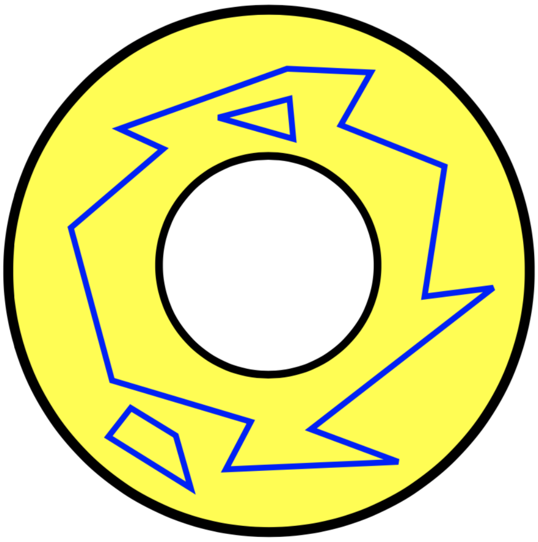
"An improved bound for the optimal Moebius band."

arXiv: 2008.11610

Definition: A T-pattern is a pair of perpendicular coplanar bend images.

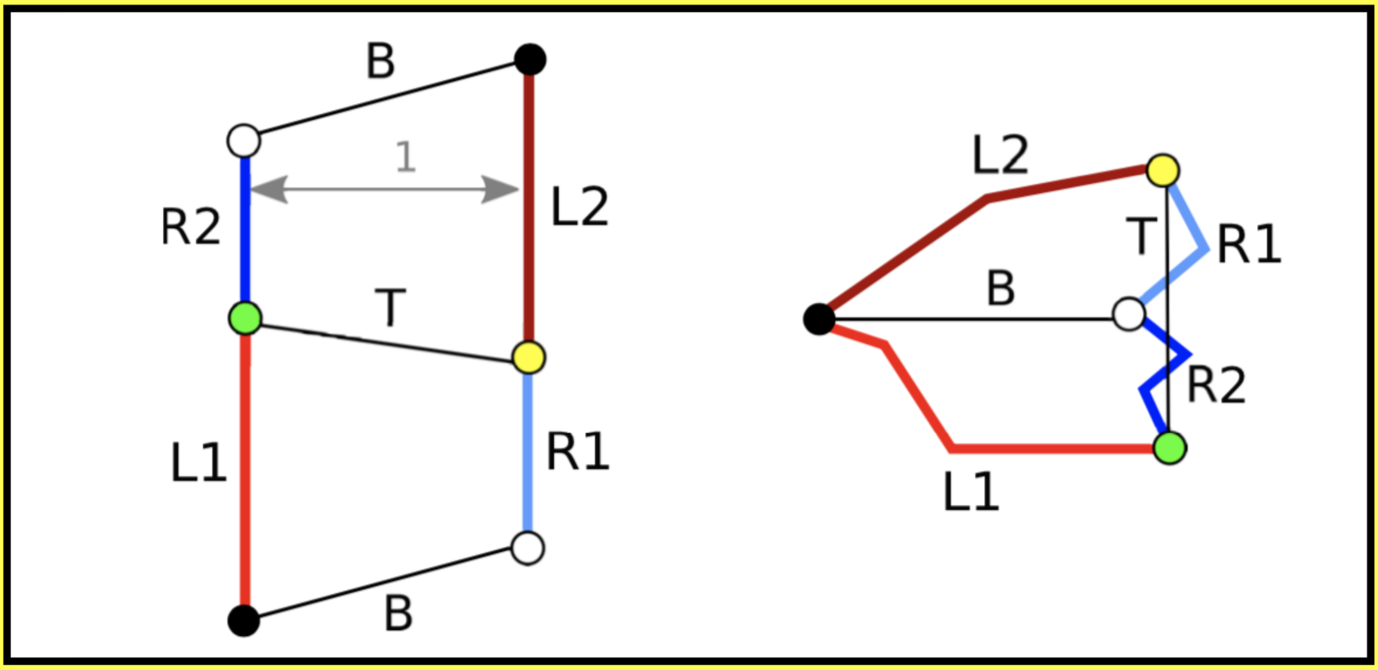
Lemma: If the aspect ratio is less than 7Pi/12. Then there exists a T-pattern.

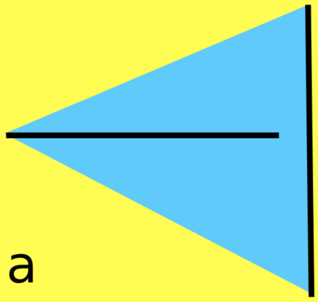
Proof sketch: For homological reasons, there is a pair of perpendicular bend images (a,b) which can be continuously connected to (b,a).



Every pair defined a unique pair of parallel planes containing them.
These planes never contain vertical lines by the aspect ratio bound.
Since they switch positions, they much coincide at some point.

cut open along the T pattern

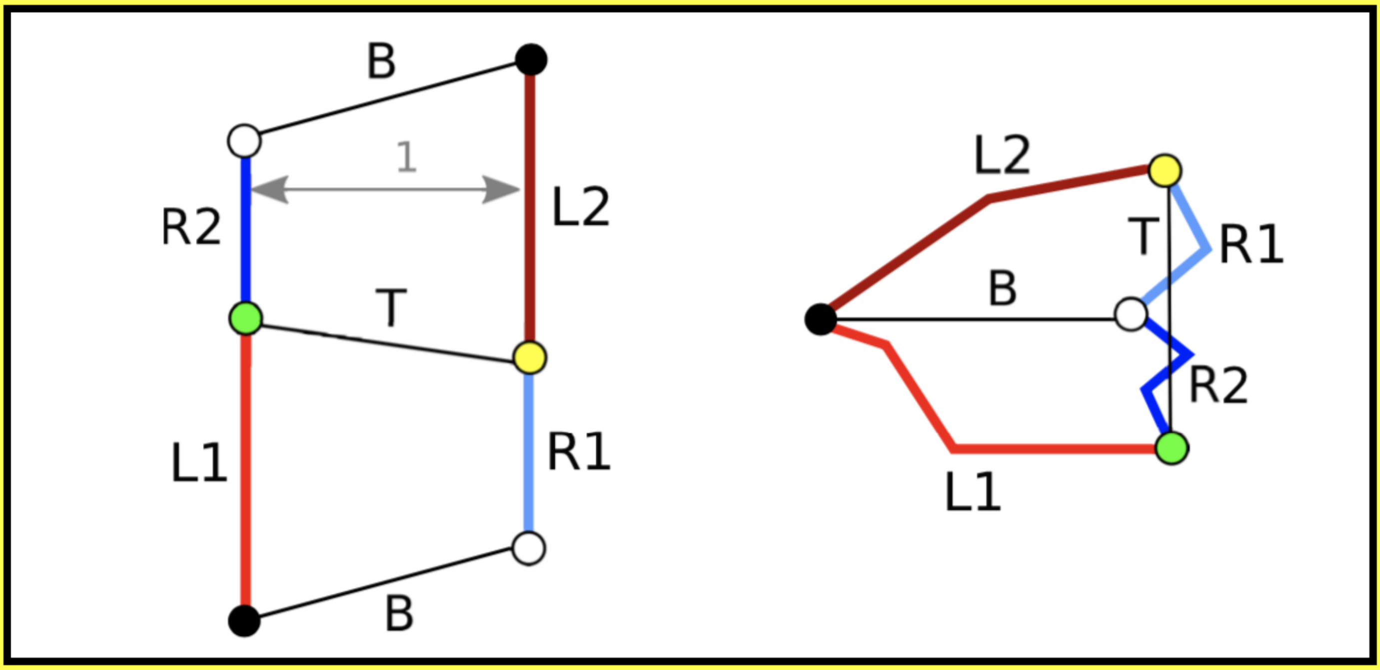




A cheap improvement:

The convex hull of the T-pattern is a
triangle of base and height at least 1.
Hence it has perimeter at least 2x(golden ratio).
But then the aspect ratio
is at least the golden ratio.

Sketch of the better bound.



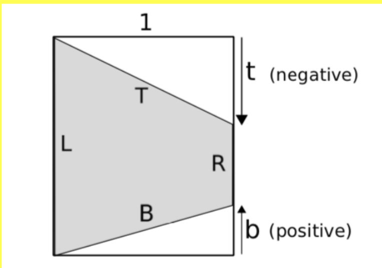
$$R_1 + R_1 \geq T,$$

$$L_1 + L_2 \geq \sqrt{B^2 + (T/2 - \epsilon)^2} + \sqrt{B^2 + (T/2 + \epsilon)^2} \geq 2\sqrt{B^2 + T^2/4}.$$

R = (R1+R1)/2 and L = (L1+L2)/2

$$2R \geq T, \qquad L \geq \sqrt{B^2 + T^2/4}.$$

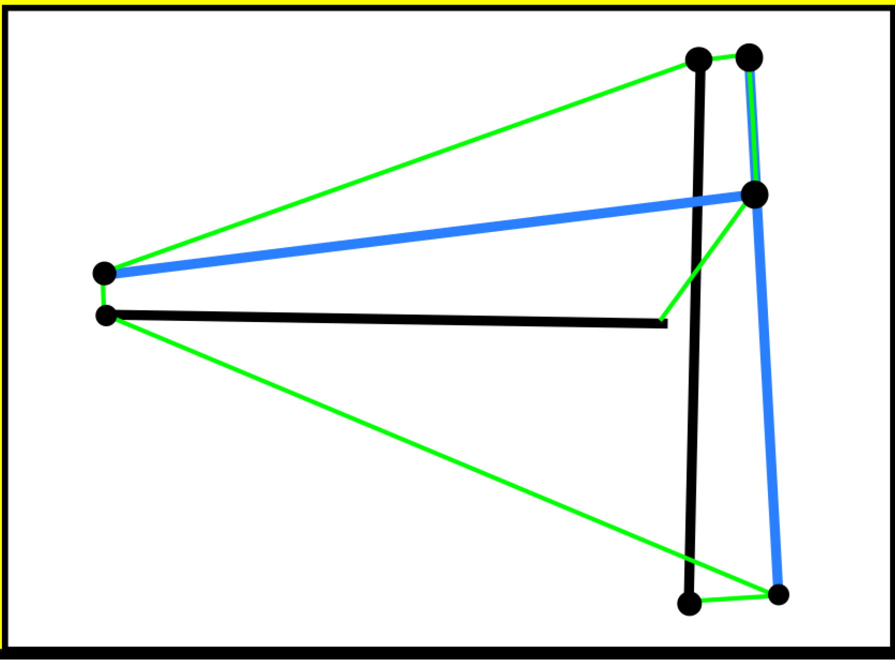
Solving this constrained optimizaton problem gives the advertised bound.



Theorem 2:
Modulo showing that 10 semi-algebraic functions are non-negative, both conjectures are true.

For comparison: My solution of the 5-electron problem did the same thing for functions on the cube $[0,1]^7$

Main ideas in the proof:
1. Topology Lemma: Forcing of "stretched ladders"
2. Geometry Lemma: Capacity of "stretched ladders"



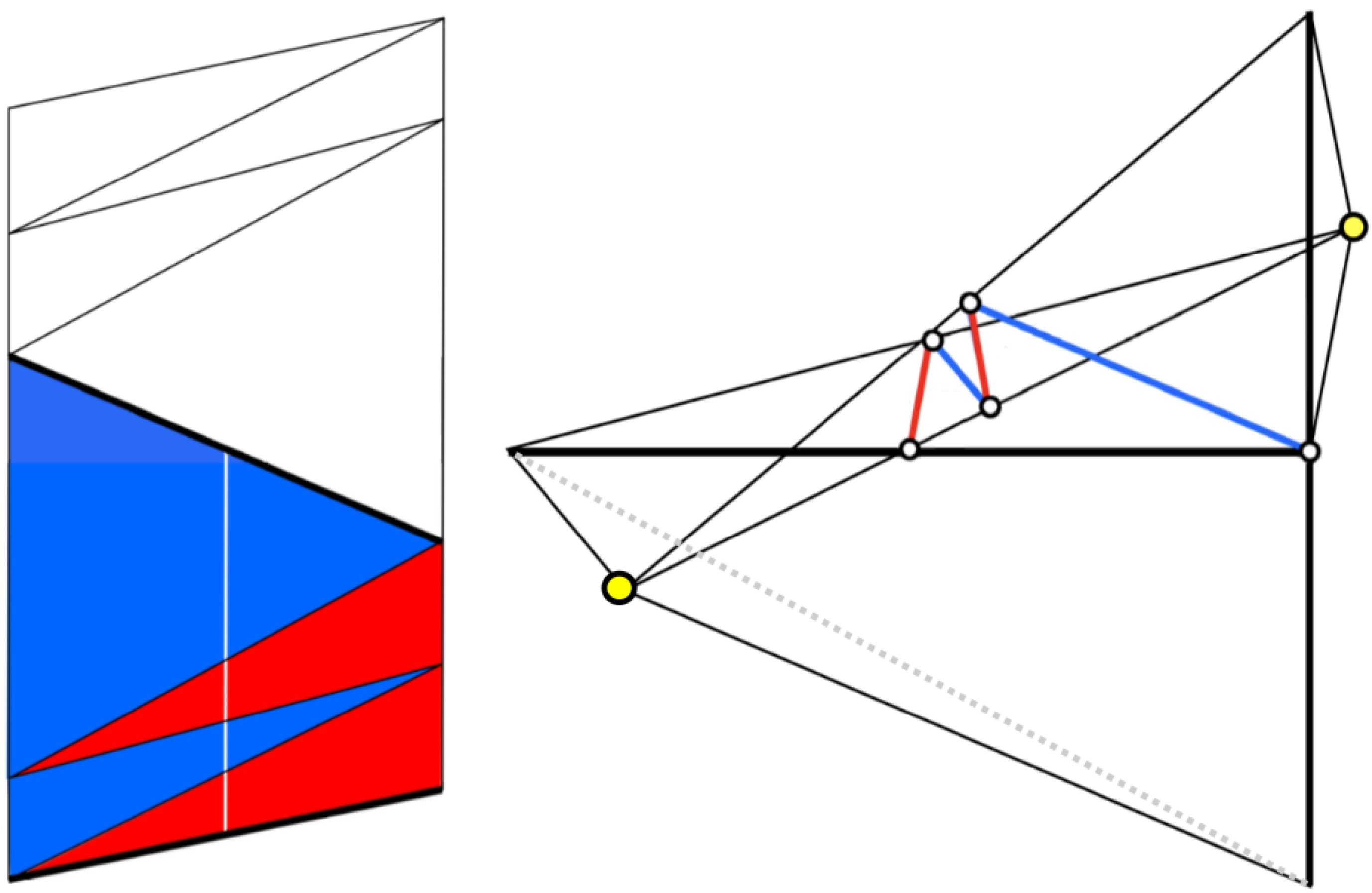
Capacity of a quad:

capacity of Y = smallest L+R which makes the map f possitile.

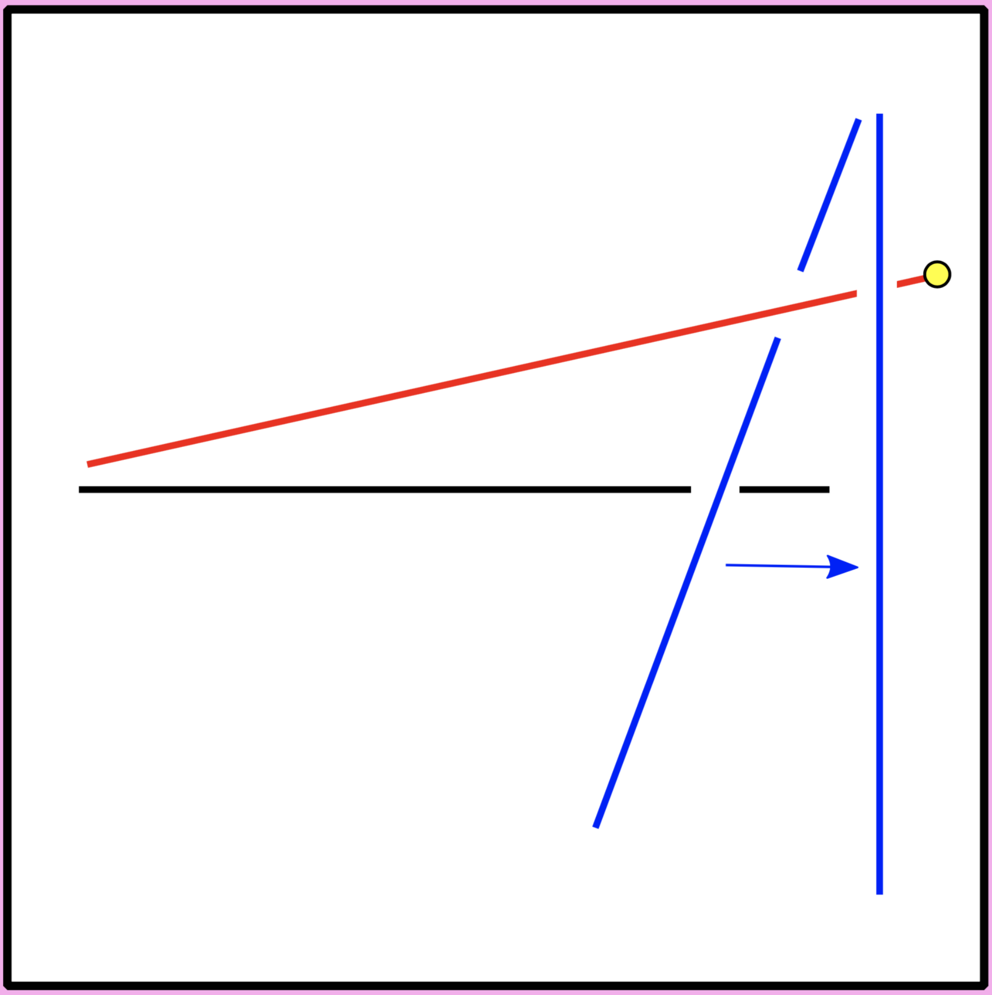
Capacity of a compound ladder:

sum of individual pieces: (L1+R1)+(L2+R2)

Topology Lemma Idea: If the aspect ratio of one half is small, bumps are created! The other half then has to get around those bumps one way or another.

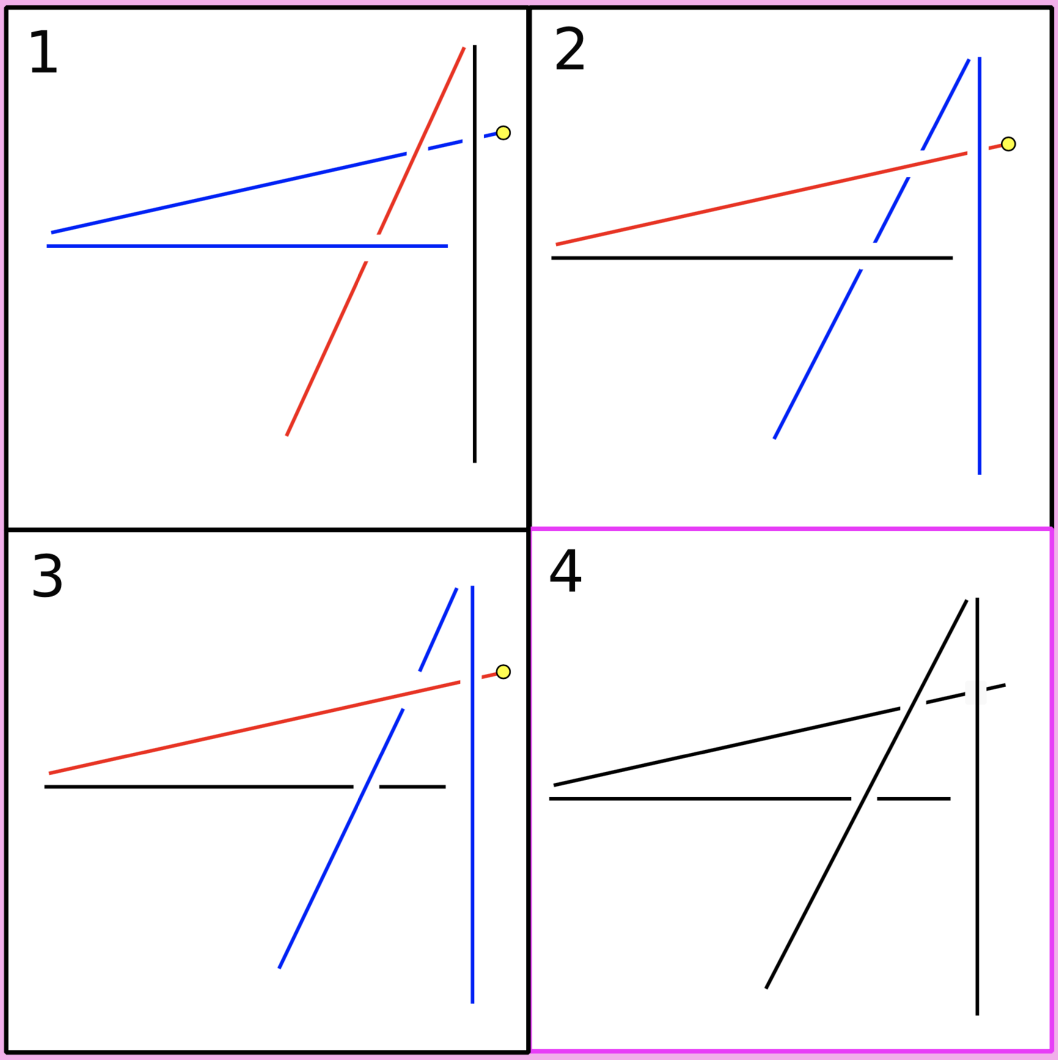


Topology lemma proof:



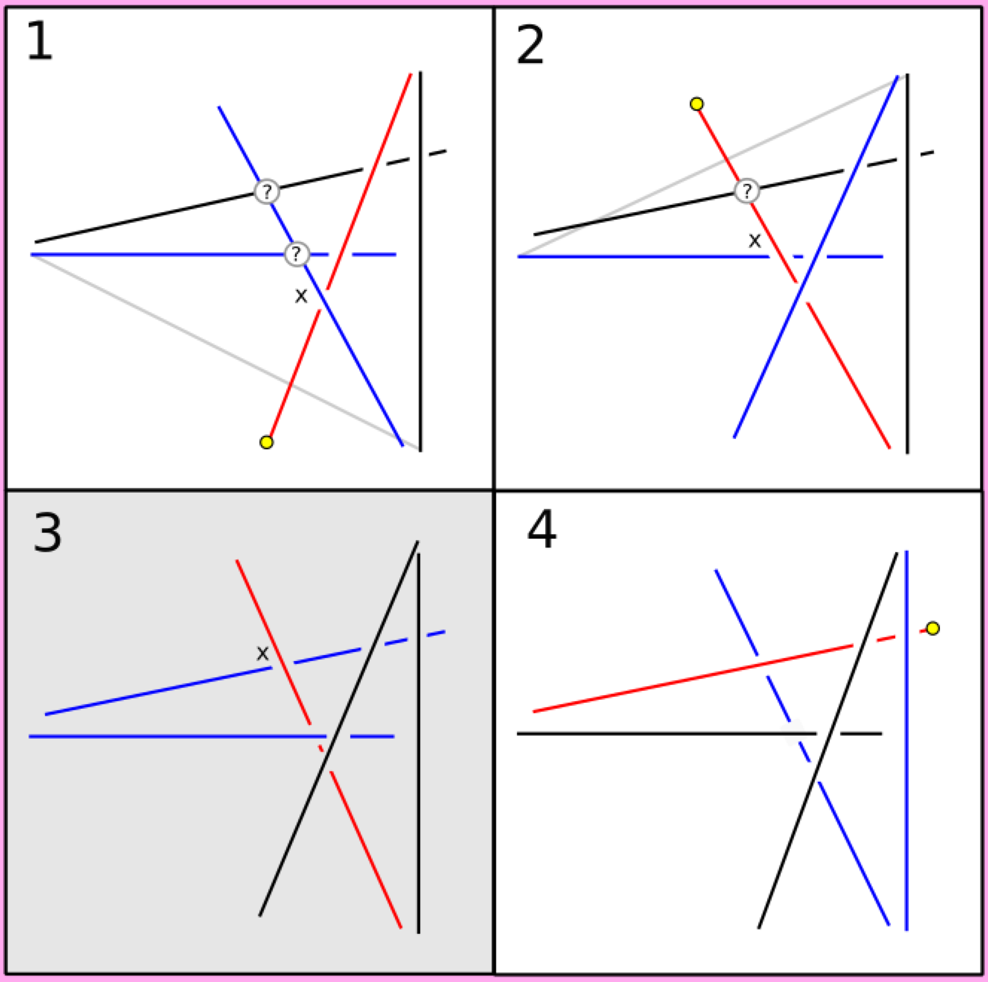
1. Modulo some geometric bounds,
incompatible crossings lead to a contradiction.

Topology lemma proof:



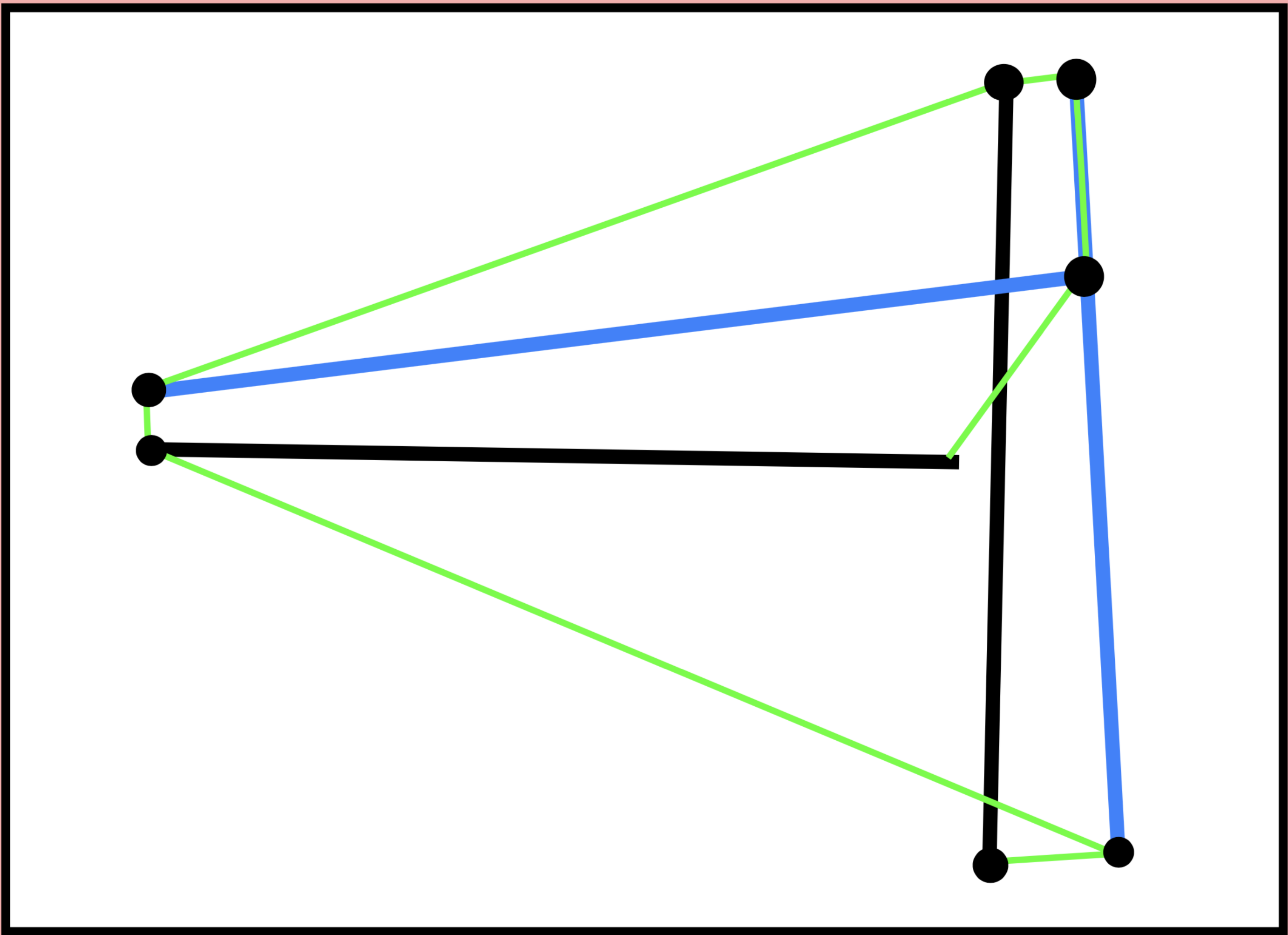
2. Use the incompatibility principle repeatedly in a case by case analysis..

Topology lemma proof:



3. All possibilities lead to stretched ladders. QED.

Geometry Lemma Proof:



Evidence: Stochastic hill climbing, simulated annealing...