SRS
SwissMAP Research
Station in Les Diablerets

RDLH
RECENT DEVELOPEMENTS IN LINK HOMOLOGY

2022 January 31st – February 4th

1 Schedule

All times are CET (GMT+1).

	Monday 31/01	Tuesday 01/02	Wednesday 02/02	Thursday 03/02	Friday 04/02
9:00 – 9:50		Sussan	Lewark		Vaz
10:00 – 10:50	Blanchet	Wagner	Stošić		Rasmussen
11:00 – 11:50			Mellit		
14:00 – 15:00				Wedrich	
17:00 – 17:50	Petkova	Gukov	Khovanov	Gonzales	
18:00 – 18:50	Lipshitz	Manolescu	Gorsky	Cautis	

2 Abstracts

Christian Blanchet (Monday 9:00 – 9:50)
Heisenberg homology of surface configurations
We study the action of the mapping class group of $\Sigma = \Sigma_{g,1}$ on the homology of configuration spaces with coefficients twisted by the discrete Heisenberg group $H = H(\Sigma)$. We will specialise to the Schrödinger representation and its finite dimensional analogue in which case we obtain representations of a central extension of the mapping class group.

Sabin Cautis (Thursday 18:00 – 18:50)
Vertex operators in knot homology
Vertex operators can inspire some fairly explicit constructions of homological knot invariants. To explain this statement we survey some categorical constructions from the past 10-15 years.

Nicolle Gonzalez (Thursday 17:00 – 17:50)
A skein theoretic Carlsson–Mellit algebra
The shuffle theorem gives a combinatorial formula for the Frobenius character of the space of diagonal harmonics in terms of certain symmetric functions indexed by Dyck paths. In their proof, Carlsson and Mellit introduce a new interesting algebra denoted $A_{q,t}$. This algebra arises as an extension of the affine Hecke algebra by certain raising and lowering operators and acts on

the space of symmetric functions via certain complicated plethystic operators. Afterwards Carlsson, Mellit, and Gorsky showed this algebra and its representation could be realized using parabolic flag Hilbert schemes and in addition to containing the generators of the elliptic Hall algebra. In this talk I will discuss joint work with Matt Hogancamp where we construct skein theoretic formulations of the representations of $A_{q,t}$ that arise in the proofs of the shuffle theorems and how this framework enables difficult computations to become simple diagrammatic manipulations as well as sheds light on potential applications to combinatorics and link homology.

Eugene Gorsky (Wednesday 18:00 – 18:50)
The Trace of the affine Hecke category
Morton and Samuelson related the skein algebra of the torus to a certain specialization of the elliptic Hall algebra. I will report on progress towards the categorification of their result, replacing the skein algebra by the horizontal trace of the affine Hecke category. This is a joint work with Andrei Negut.

Sergei Gukov (Tuesday 17:00 – 17:50)
Decorated TQFTs: from q-series to non-semisimple invariants
Invariants of knots and 3-manifolds in Quantum Topology are usually constructed from basic ingredients of algebraic origin. Such algebraic structures, in turn, describe symmetries of integrable lattice models (as in the case of quantum groups) or vertex operator algebras (as in the case of tensor categories). In simple instances, as e.g. in the WRT invariants of 3-manifolds and in the corresponding knot invariants, these symmetries (algebraic structures) do not have interesting symmetries of their own, i.e. automorphisms. When they do, however, the cutting-and-gluing rules in the corresponding TQFT become more interesting and non-trivial. The goal of this talk, based on the recent work with François Costantino and Pavel Putrov `arXiv:2107.14238`, is to describe a unified framework ("decorated TQFT") that can put under the same roof additional structures / decorations that appear e.g. in the study of G-crossed modules, sutured and bordered Floer homology, $\widehat{Z}$ and BCGP invariants, and various other TQFT-like structures that until recently have been developed independently. In other words, the goal is to build new bridges that can facilitate translation between different ways decorations are described in these theories, thereby allowing to explore relations among them and building new decorated TQFTs.

Mikhail Khovanov (Wednesday 17:00 – 17:50)
Topological theories and automata
We'll explain the connection between topological theories for one-manifolds with defects and values in the Boolean semiring and automata and their generalizations. A finite state automaton accepts a regular language. To each pair of a regular language and a circular regular language we associate a topological theory for one-dimensional manifolds with zero-dimensional defects labelled by letters of the language. This theory takes values in the Boolean semiring B. Universal construction gives rise to a monoidal category of B-semilinear combinations of one-dimensional cobordisms with defects modulo skein relations. The latter category can be interpreted as a semilinear rigid monoidal closure of standard structures associated to a regular language, including minimal deterministic and nondeterministic finite state automata for the language and the syntactic monoid. The circular language plays the role of a regularizer, allowing to define the rigid closure of these structures. The talk is based on a joint paper in progress with Mee Seong Im.

Lukas Lewark (Wednesday 9:00 – 9:50)
Khovanov homology and rational unknotting
In this talk, we will see a new geometric application of Khovanov homology, generalizing work by Alishahi and Dowlin. We'll use a universal Khovanov homology theory that associates to a knot diagram a $\mathbb{Z}[G]$-complex C, where G is a formal variable, such that $C/(G = 1)$ has homology $\mathbb{Z}$. We'll define a metric on homotopy classes of such $\mathbb{Z}[G]$-complexes. This metric turns out to provide a lower bound for the proper rational unknotting number, i.e. the minimal number of connectivity preserving rational tangle replacements needed to make a knot trivial. This talk is based on joint work with Damian Iltgen and Laura Marino (see `arXiv:2110.15107`).

Robert Lipshitz (Monday 17:00 – 17:50)
Khovanov homology and non-orientable surfaces
The first half of the talk will introduce an invariant of non-orientable knot cobordisms, coming from deformed Khovanov homology, imitating the construction of the Heegaard Floer mixed invariant (i.e., the Seiberg–Witten invariant). The second half will discuss properties and computations of Khovanov homology invariants of cobordisms, including leveraging work of Hayden–Sundberg to show that they distinguish some exotic pairs of non-orientable surfaces. This is joint with Sucharit Sarkar.

Ciprian Manolescu (Tuesday 18:00 – 18:50)

A knot Floer stable homotopy type

Given a grid diagram for a knot or link K in $\mathbb{S}^3$, we construct a spectrum whose homology is the knot Floer homology of K. We conjecture that the homotopy type of the spectrum is an invariant of K. Our construction does not use holomorphic geometry, but rather builds on the combinatorial definition of grid homology. We inductively define models for the moduli spaces of pseudo-holomorphic strips and disk bubbles, and patch them together into a framed flow category. The inductive step relies on the vanishing of an obstruction class that takes values in a complex of positive domains with partitions. (This is joint work with Sucharit Sarkar.)

Anton Mellit (Tursday 9:00 – 9:50)

Knot homology, tautological classes and $\mathfrak{sl}_2$

I will talk about a new kind of structure we discovered on the triply graded Khovanov–Rozansky link homology, motivated by a study of character varieties. The complexes whose homology is the link homology are endowed with an action of a certain DG algebra. Applying the Koszul duality we pass to the y-ified Khovanov–Rozansky homology, which now acquires an action of a commuting family of operators we call tautological classes. Lefschetz property satisfied by the second tautological class implies the symmetry conjectured by Dunfield, Gukov and Rasmussen. This is a joint work with Eugene Gorsky and Matt Hogancamp

Ina Petkova (Monday 17:00 – 17:50)

Annular link Floer homology and $\mathfrak{gl}(1|1)$

The Reshetikhin–Turaev construction for the quantum group $U_q(\mathfrak{gl}(1|1))$ sends tangles to $\mathbb{C}(q)$-linear maps in such a way that a knot is sent to its Alexander polynomial. Tangle Floer homology is a combinatorial generalization of knot Floer homology which sends tangles to (homotopy equivalence classes of) bigraded dg bimodules. In earlier work with Ellis and Vertesi, we show that tangle Floer homology categorifies a Reshetikhin–Turaev invariant arising naturally in the representation theory of $U_q(\mathfrak{gl}(1|1))$; we further construct bimodules E and F corresponding to E, F in $U_q(\mathfrak{gl}(1|1))$ that satisfy appropriate categorified relations. After a brief summary of this earlier work, I will discuss how the horizontal trace of the E and F actions on tangle Floer homology gives a $\mathfrak{gl}(1|1)$-action on annular link Floer homology that has an

interpretation as a count of certain holomorphic curves. This is based on joint work in progress with Andy Manion and Mike Wong.

Jacob Rasmussen (Friday 10:00 – 10:50)
Knot Floer homology, sutures, and the solid torus
Over the last few years there has been a lot of progress on categorified invariants for knots in the solid torus. In the hope that comparison with knot Floer homology may still be a fruitful exercise, I'll give a graphical description of $\widehat{HFK}$ for such knots. This is an extension of earlier work with Jonathan Hanselman and Liam Watson.

Marko Stošić (Wednesday 10:00 – 10:50)
Knot invariants, knot complements and quivers
In this talk I will give an overview of the knots-quivers correspondence, from the first formulation till some of the recent results. Originally knots-quivers correspondence was introduced by rewriting colored HOMFLY-PT invariants in the form of quiver generating series for suitable quivers. Recently, other knot invariants have also been shown to be re-writable in a "quiver form", namely Gukov–Manolescu $\widehat{Z}$-invariants of knot complements. I will present some basic facts about these correspondences, as well as relations between them.

Joshua Sussan (Tuesday 9:00 – 9:50)
p-DG structures in higher representation theory
One of the goals of the categorification program is to construct a homological invariant of 3-manifolds coming from the higher representation theory of quantum groups. The WRT 3-manifold invariant uses quantum groups at a root of unity. p-DG theory was introduced by Khovanov as a means to categorify objects at prime roots of unity. We will review this machinery and show how to construct categorifications of certain representations of quantum $\mathfrak{sl}_2$ at prime roots of unity.

Pedro Vaz (Friday 9:00 – 9:50)
2-Verma modules and link homologies
In this talk I will explain how 2-Verma modules can be used to produce several link homologies for knots in the 3-space and in the solid torus. The various

constructions go through a categorification of (parabolic) Verma modules and its tensor products with finite-dimensional irreducibles of $\mathfrak{gl}(n)$. The material presented spreads along several collaborations with Abel Lacabanne, and Grégoire Naisse.

Emmanuel Wagner (Tuesday 10:00 – 10:50)
Categorification of colored Jones polynomial at root of unity
In the first part of the talk, we will define colored symmetric $\mathfrak{gl}(2)$ homologies. These are homological link invariants. We'll focus on a combinatorial and elementary approach of this construction and explain how it is related to Soergel bimodules and to triply graded homology. In the second part, we'll see how to endow them with p-DG structures and providing so a categorification of colored Jones polynomial at root of unity. This is a joint work with You Qi, Louis-Hadrien Robert and Joshua Sussan.

Paul Wedrich (Thursday 14:00 – 15:00)
A skein relation for singular Soergel bimodules
Soergel bimodules categorify Hecke algebras and lead to invariants of braids that take values in monoidal triangulated categories. In this process, the quadratic "skein relation" on Artin generators is promoted to a distinguished triangle. I will talk about an analog of this relation in the setting of singular Soergel bimodules and Rickard complexes, in which the distinguished triangle gets replaced by a longer one-sided twisted complex. Joint work with M. Hogancamp and D.E.V. Rose.